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Investigating incomplete mixing models in cross junctions under real-world conditions of water distribution networks

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ABSTRACT

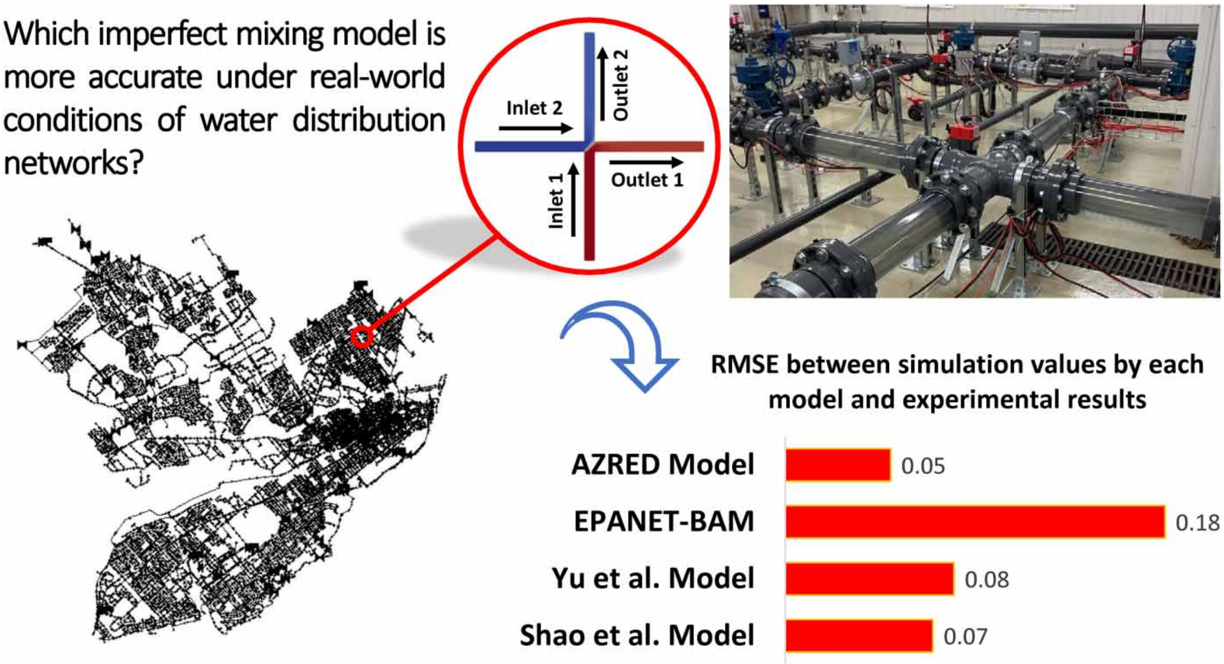
Questions have been raised about the correctness of water quality models with complete mixing assumptions in cross junctions of water distribution systems. Recent developments in the mixing phenomenon within cross junctions of water distribution networks (WDNs) have heightened the need for evaluating the existing incomplete mixing models under real-world conditions. Therefore, in this study, two cross junctions with pipe diameters of 100 × 100 × 100 × 100 mm and 150 × 150 × 150 × 150 mm were employed in laboratory experiments to evaluate six existing incomplete mixing models for 25 flow rate scenarios ranging between 1.5 and 3.0 L/s. It was observed that within the same flow rate scenario, the degree of mixing in a cross junction with a pipe relative roughness of 6.00×10^{-5} (pipe diameter of 25 mm) was higher than that in a cross junction with a pipe relative roughness of 3.00×10^{-5} (pipe diameter of 50 mm) and smaller. Considering the real-world size of pipes in evaluating the incomplete mixing models showed that two incomplete mixing models, AZRED and the one by Shao *et al.*, had the best accordance with the results of the laboratory experiments.

Key words: complete mixing assumption, imperfect mixing models, water distribution networks, water quality models

HIGHLIGHTS

- Laboratory experiments were conducted under real-world conditions to assess models.
- Two pipe diameters of 100 and 150 mm were investigated in the laboratory.
- For junction sizes less than 100 mm, mixing is not equivalent under the same flow.
- For junction sizes greater than 100 mm, mixing is equivalent under the same flow.
- AZRED and Shao *et al.* models better reproduce observations than other models.

GRAPHICAL ABSTRACT



INTRODUCTION

Modeling urban water distribution networks (WDNs) is useful for providing drinking water of appropriate quantity, quality, security, and cost. Accordingly, developments are continuously ongoing to increase the accuracy and efficiency of hydraulic and quality models of urban WDNs. Current water quality models have been questioned due to the simplifying assumption of complete mixing at cross junctions (Orear *et al.* 2005; Austin *et al.* 2008; Ho 2008; Song *et al.* 2009; Romero-Gomez *et al.* 2011; Shang *et al.* 2021). Therefore, many numerical and experimental studies have provided important insights into the mixing at cross junctions of WDNs. Besides, some empirical and mechanistic water quality models have been developed considering incomplete mixing (IM).

Some laboratory studies on the mixing phenomenon have been undertaken in cross junctions with pipe diameters of 10–50 mm (pipe relative roughness range of 1.50×10^{-4}–3.00×10^{-5} if assuming a pipe roughness of 0.0015 mm for poly vinyl chloride (PVC) pipes (Dahl *et al.* 1990)) (Austin *et al.* 2008; Choi *et al.* 2008; Ho & Khalsa 2008; Shao *et al.* 2014). In these laboratory investigations, the Reynolds number (Re) of flow in the inlets and outlets, which was the primary variable of the experiments, was in the range of 10,000–40,000 (Re number is the ratio of the inertial to viscous forces).

Besides considering the mixing phenomenon at cross junctions within turbulent flow, some researchers have also investigated the mixing phenomenon for laminar and transitional flows (where the flow is between the characteristics of laminar and turbulent flow) ($500 < Re < 5,000$) within the pipe diameter range of 15–50 mm (the pipe relative roughness range of 1.50×10^{-4}–3.00×10^{-5}) (McKenna *et al.* 2008; Romero-Gomez & Choi 2011; Braun *et al.* 2014; van Summeren *et al.* 2017; Shao *et al.* 2019; Sun *et al.* 2022). These authors have also visually analyzed the mixing within cross junctions in the laboratory to capture more details.

Orear *et al.* (2005) also found that the degree of mixing (which is defined as the ratio of the greatest outlet concentration to the smallest outlet concentration) in a cross junction with pipe diameters of $10 \times 10 \times 10 \times 10$ mm (pipe relative roughness of 1.50×10^{-4}) was higher than that with pipe diameters of $50 \times 50 \times 50 \times 50$ mm (relative roughness of 0.30×10^{-4}) when the Re numbers in all the pipes of the two cross junctions were around 43,000.

For laminar flow ($Re < 2,000$), Shao *et al.* (2019) also showed that the larger the pipe diameter, the higher the degree of mixing for the same Re. In another study, Yousefian & Duchesne (2022a) experimentally studied the behavior of mixing under high pressure within two pipe diameters of 100 and 150 mm (relative roughness of 1.50×10^{-5} and 1.00×10^{-5}). The authors found that the mixing in cross junctions with two pipe diameters of 100 and 150 mm (relative roughness of

1.50×10^{-5} and 1.00×10^{-5}) and under the pressures of 5, 140, 320, and 430 kPa at the cross junctions has a similar degree of mixing when the flow rates are similar.

Further details on studies and existing IM models in this field are summarized in Yousefian & Duchesne (2022b). According to this review, currently there are no studies conducted to evaluate and compare these models under real-world conditions, despite all the available IM models. For this purpose, in the current research, the pipe diameter and flow rate conditions of one WDN in North America were investigated and applied within the laboratory to evaluate the accuracy of the following six existing IM models under real-world conditions: AZRED (Austin *et al.* 2008), EPANET-BAM (Ho & Khalsa 2008), IM model by Yu *et al.* (2014), IM model by Shao *et al.* (2014), IM model by Hernández Cervantes *et al.* (2021), and IM model by Hammoudi (2021). The main objective of this study was to identify the model that best represents the degree of mixing under realistic flow conditions within cross junctions with real-world pipe diameters.

METHODOLOGY

This study consisted of five main steps (see Figure 1). First, a statistical analysis of a real-world WDN in North America was conducted to find the most common cross-junction pipe diameters and flow rates. The WDN in the laboratory was then redesigned and rebuilt based on these specifications (found in the previous statistical analysis) in order to make the subsequent experimental analyses. In other words, the diameter of the pipes and the flow rates tested in the laboratory were set based on

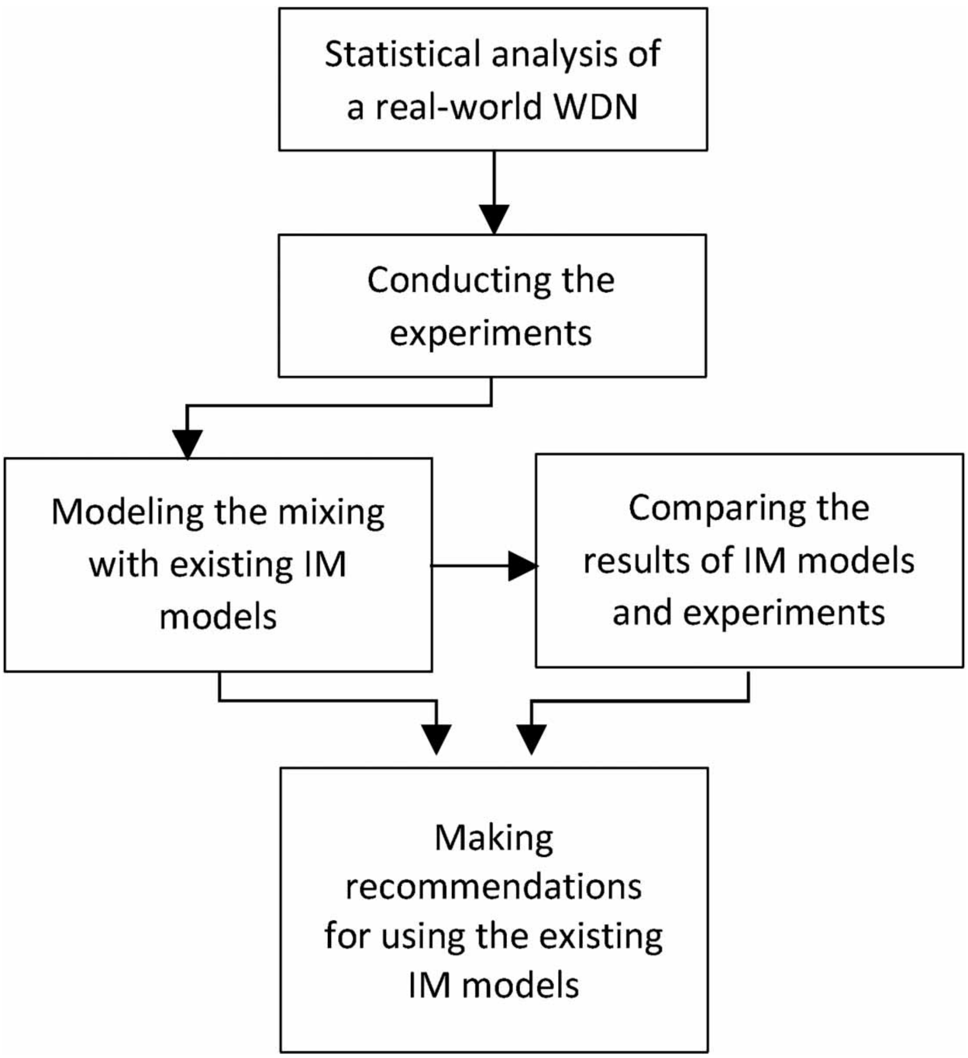


Figure 1 | Flow chart of the steps taken in the current research.

the results of the statistical analysis of the real-world WDN. The tested existing IM models were coded and applied to model mixing under the same conditions as the laboratory experiments. Finally, some recommendations were made for using the existing IM models.

Existing IM models

As explained in Yousefian & Duchesne (2022b), there are different empirical and mechanistic models to compute mixing in cross junctions. Among these models, six most recent IM models were selected, and their outlines are given below. More details about the models are given in the Supplementary Material.

AZRED

AZRED is the first IM model that can project the solute concentration in cross-junction outlets based on the inlet concentration ratio (the ratio of the concentration of the highest concentration inlet to the concentration of the lowest concentration inlet), the ratio of inlet Re numbers, and the ratio of outlet Re numbers (Austin *et al.* 2008). Austin *et al.* (2008) conducted experiments with cross junctions made of pipe diameters of $25 \times 25 \times 25 \times 25$ mm and Re numbers ranging from 10,000 to 42,000 to develop their IM model (AZRED) and used extrapolation for Re values greater than 42,000. They mentioned that AZRED cannot be used in cases of differing pipe diameters in a cross junction and with angles that differ from 90° between the pipes in the junction (Austin *et al.* 2008; Choi *et al.* 2008).

EPANET-BAM

Ho (2008) proposed a mechanistic IM model to study mixing in cross junctions. The author considered only the advection terms of the transport equation to develop an IM model and focused on the inlet and outlet Re number ratios as the effective parameters of the mixing phenomenon. In this model, only cross junctions with equal pipe diameters were considered, and experimental data from Romero-Gomez *et al.* (2008) and McKenna *et al.* (2007) were used to validate this IM model for cross junctions. Ho (2008) named his model EPANET-BAM, derived from the bulk-advection method.

Ho (2008) found that this bulk-mixing model can be used to compute the lower and upper bounds for the concentration resulting from the mixing phenomenon, alongside the complete mixing. C_{bulk} and C_{complete}. C_{bulk} of each outlet is used by Ho (2008) in combination with C_{complete} to compute modified concentrations at that outlet (C_{combined}) using a scaling parameter (s) determined from experimental results for each flow case and junction configuration. This formula is expressed in the following equation:

$$C_{\text{combined}} = C_{\text{bulk}} + s(C_{\text{complete}} - C_{\text{bulk}}) \tag{1}$$

Ho (2008) selected the scaling parameter values of 0.00, 0.20, 0.50, 0.80, and 1.00 to compare the results of the model for each of these values, with the experimental data of Romero-Gomez *et al.* (2008). He observed that the scaling parameter value of 0.50 provided the best agreement between the simulated and observed concentrations when the Re number of all the inlets and outlets is between 5,000 and 50,000. Likewise, Ho & Khalsa (2008) held the view that the results of EPANET-BAM showed an acceptable agreement with the numerical and experimental results when the scaling parameter value was 0.50. However, they still believed it necessary to calibrate the scaling parameter for more accurate results. Besides, since in the real world, there is no sufficient data to tune the scaling parameter for all cross junctions, a scaling parameter of 0.50 was considered for all the flow rate scenarios modeled with EPANET-BAM in the current study.

Analytical IM model by Yu *et al.* (2014)

Yu *et al.* (2014) experimentally and numerically investigated the mixing phenomenon within cross junctions with different pipe diameters. The ANSYS Fluent software was employed for the numerical simulations, NaCl was utilized as a tracer, and the pipe diameters were 25, 32, and 50 mm (with pipes relative roughness of 0.60×10^{-4}, 0.46×10^{-4}, and 0.30×10^{-4}, respectively), and the pressure was about 55 kPa in the experiments of Yu *et al.* (2014). Based on their experimental and numerical results, Yu *et al.* (2014) proposed a new mixing index and a regression equation to compute the degree of mixing as a function of the inlet pipe diameter ratio (the ratio of the pipe diameter of the highest concentration inlet to the pipe diameter of the lowest concentration inlet) and of the inlet and outlet Re number ratios.

Analytical IM model by Shao *et al.* (2014)

Shao *et al.* (2014) suggested an analytical solution for the mixing phenomenon in cross junctions with two configurations: opposing inlets (when two inlets of the cross junction are in front of each other) and adjacent inlets (where two inlets in the cross junction are next to each other). As in Ho & O'Rear Jr. (2009), their laboratory experiments demonstrated that mixing is almost complete in the case of opposing inlets. For the adjacent inlets, Shao *et al.* (2014) investigated two possible flow patterns: higher momentum in the opposing pipes or higher momentum in the adjacent pipes. To take this into account, they defined a new flow distribution factor to estimate the solute concentration in the outlets. To realize their laboratory experiments, they used a cross junction with equal pipe diameters of 25 mm under a Re range of 15,700–90,000. This analytical model was also validated using the $k\varepsilon$ turbulent model of Ansys Fluent software, with an unstructured grid of more than 2.63 million cells and a mesh size of 2.5 mm for the pipe and 1.5 mm for the junction.

Analytical IM model by Hernández Cervantes *et al.* (2021)

Hernández Cervantes *et al.* (2021) proposed polynomial equations for imperfect mixing in cross junctions. In this IM model, the ratio of flow rates in the inlets (the ratio of flow rate in the highest concentration inlet to the flow rate in the lowest concentration inlet), the ratio of flow rates in the outlets (the flow rate in the outlet adjacent to the highest concentration inlet divided by the flow rate in the outlet opposing the highest concentration inlet), and the ratio of concentrations in the inlets were considered as the independent variables, while the ratio of concentrations in the outlets was the dependent variable. Accordingly, 12 flow rate scenarios were selected and the solute concentrations in the outlets were measured. For each of the 12 flow rate scenarios, a polynomial equation was fitted to predict the ratio of solute concentrations in the outlets. Then, to find the ratio of concentrations in the outlets for a specific flow rate scenario not studied by the authors, the characteristics of the most similar flow rate scenario should be chosen. In the final step, the polynomial equation of the chosen flow rate scenario can be used to predict the outlet's concentration for the specific case.

Analytical IM model by Hammoudi (2021)

Hammoudi (2021) also numerically investigated the effect of Re number, pipe diameter, mixing time (which is the time that the fluid spends in the mixing area), diffusivity (which describes how rapidly the scalar quantity would move through the fluid in the absence of convection), and the difference between the density of clean and contaminated water on mixing in cross junctions. For their numerical simulations, the 3D $k\varepsilon$ and large eddy simulation models of ANSYS CFX 19.1, an element-based finite volume method, were employed within an unstructured mesh. In this research, a sensitivity analysis of the aforementioned parameters was conducted on the mixing phenomenon. Hammoudi (2021) pointed out that the density of contaminated water, the pipe diameter, and the Re number can significantly affect the degree of mixing. Therefore, a regression equation was provided to estimate the degree of mixing within cross junctions based on the ratio of flow rates in the inlets, the ratio of flow rates in the outlets, the density and diffusivity of contaminated water, the pipe diameter of the cross junction, and the Re number of the contaminated inlet.

Statistical analysis of a real WDN

The number and type of cross junctions that were assumed with IM (i.e., with two adjacent inlets), the diameter of commonly used pipes, and the most frequent flow rates were counted for a North American WDN serving a population of about half a million in order to select the pipe diameters and flow characteristics to include in the experimental set-up. The mini WDN Laboratory of Institut National de la Recherche Scientifique (INRS; see description below) was then redesigned and rebuilt based on these conditions to evaluate the six selected IM models. Since most of the existing IM models were developed based on experimental results obtained with 25 or 50-mm pipes, these pipe diameters were also considered in this study, even though they are not frequently used in real WDNs.

The most frequent conditions of real-world WDNs were extracted from the analysis of 24-h average water demand on a normal day in the selected North American city. In this analysis, all cross junctions were examined to determine if mixing was complete. To do so, the cross junctions with two adjacent inlets and two outlets were identified as incompletely mixed cross junctions. After counting the number of cross junctions with incomplete mixing, it was found that about 65% of cross junctions in this WDN have IM conditions. Moreover, in this WDN, the minimum pipe diameter is 100 mm, the most frequently used pipe diameters are 100 and 150 mm, and flow rates are between 1.50 and 3.00 L/s, which are the most common during a normal day. Therefore, two cross junctions with pipe diameters of 100 and 150 mm were selected for this study, while the tested flow rate scenarios are those listed in Table 1.

Experimental set-up

The experiments were carried out in the Mini Water Distribution Network Laboratory of the INRS in Quebec City, Canada. The network is equipped with two pumps, which are a 3 hp pump (Xylem-AquaBoost) and a 75 hp pump (Berkeley-B4EPBMS), and 12 flow control valves with electric actuators (Assured Automation-P2R4 with S4 actuator) to apply a variety of pressure and flow configurations. The network is also equipped with six pressure probes (Ashcroft-G2) and nine electromagnetic flow meters (ModMAG-M2000) (Figure 2). Sodium chloride (NaCl) was used as a soluble tracer for the laboratory experiments, and a conductivity meter (Teledyne-LXT220) was installed in each leg of the cross junction (for a total of four conductivity meters) to measure the conductivity (related to the concentration of salts) in each inlet and outlet. All the pump and valve settings were set through a central computer (Honeywell-EBI R430 and Honeywell-Controller HC900). The measurements of flow, pressure, and conductivity were recorded every 5 s. These data were averaged over 60 s intervals to collect repeatable data by reducing signal noise from the sensors.

Tap water was used throughout the whole network, and salt (NaCl) was injected into the network using a pulse (or an injection) pump, resulting in solutions with salt concentrations in the range of 100–1,000 mg/L. The distance between the salt injection point and the cross junction was about 3 m to ensure that the transverse mixing in the solution occurred entirely

Table 1 | Flow rate scenarios tested in the laboratory

Scenario	South inlet (salty water) (L/s)	West inlet (tap water) (L/s)	North outlet (L/s)	East outlet (L/s)	Inlet flow rate ratio (west/south)	Outlet flow rate ratio (east/north)
1	1.50	3.00	3.00	1.50	0.50	0.50
2	1.50	3.00	2.50	2.00	0.50	0.80
3	1.50	3.00	2.25	2.25	0.50	1.00
4	1.50	3.00	2.00	2.50	0.50	1.25
5	1.50	3.00	1.50	3.00	0.50	2.00
6	2.00	2.50	3.00	1.50	0.80	0.50
7	2.00	2.50	2.50	2.00	0.80	0.80
8	2.00	2.50	2.25	2.25	0.80	1.00
9	2.00	2.50	2.00	2.50	0.80	1.25
10	2.00	2.50	1.50	3.00	0.80	2.00
11	2.25	2.25	3.00	1.50	1.00	0.50
12	2.25	2.25	2.50	2.00	1.00	0.80
13	2.25	2.25	2.25	2.25	1.00	1.00
14	2.25	2.25	2.00	2.50	1.00	1.25
15	2.25	2.25	1.50	3.00	1.00	2.00
16	2.50	2.00	3.00	1.50	1.25	0.50
17	2.50	2.00	2.50	2.00	1.25	0.80
18	2.50	2.00	2.25	2.25	1.25	1.00
19	2.50	2.00	2.00	2.50	1.25	1.25
20	2.50	2.00	1.50	3.00	1.25	2.00
21	3.00	1.50	3.00	1.50	2.00	0.50
22	3.00	1.50	2.50	2.00	2.00	0.80
23	3.00	1.50	2.25	2.25	2.00	1.00
24	3.00	1.50	2.00	2.50	2.00	1.25
25	3.00	1.50	1.50	3.00	2.00	2.00

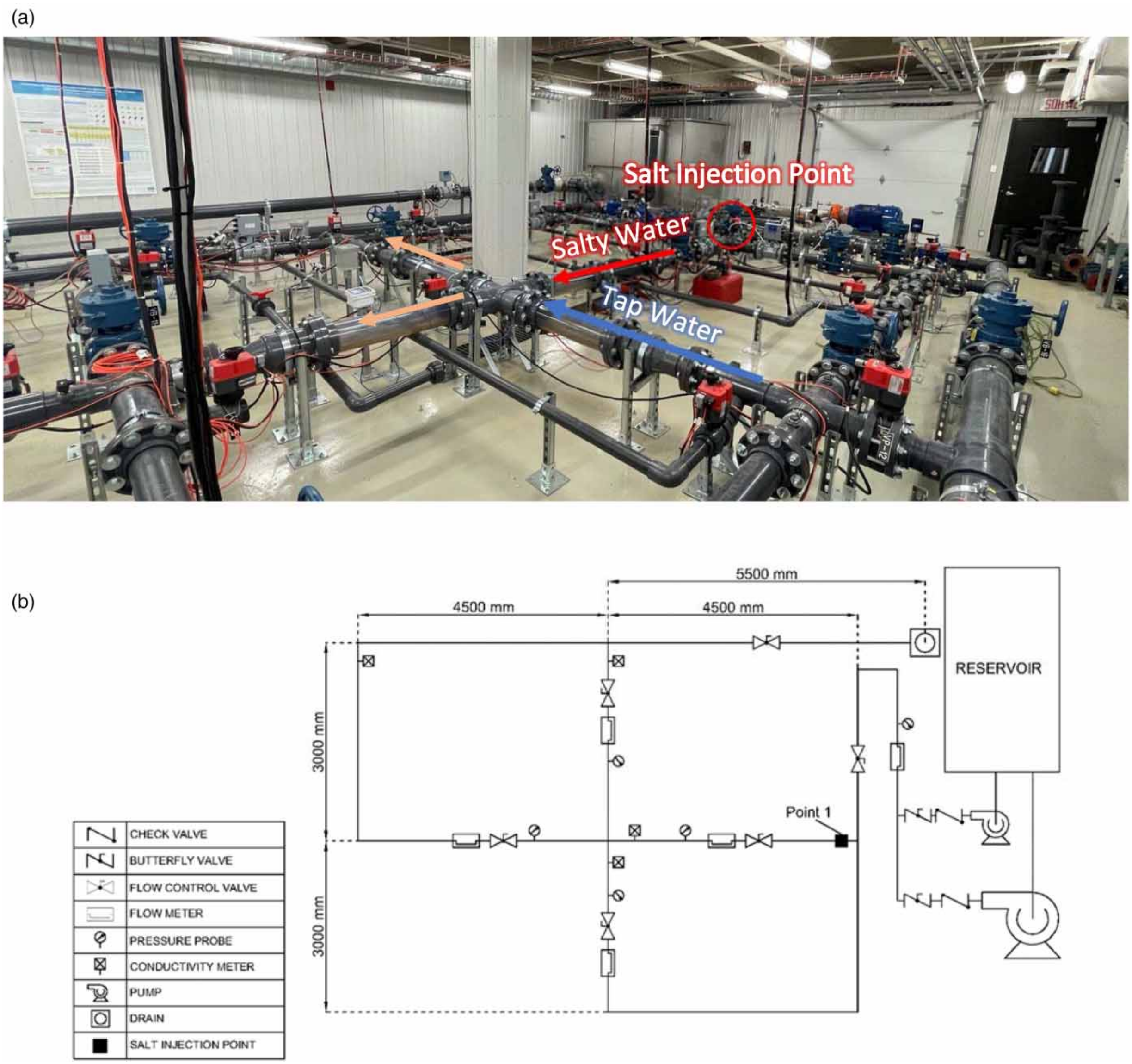


Figure 2 | Laboratory experiments to obtain the dimensionless concentrations: (a) schematic picture of the laboratory experimental set-up and (b) the laboratory experimental set-up plan.

before reaching the cross junction (this was verified by measuring conductivity at different depths of the pipes). In all laboratory experiments, the salt was injected into the southern pipe (red arrow in Figure 2(a)), tap water was supplied from the western pipe (blue arrow in Figure 2(a)), and northern and eastern pipes were the outlets (orange arrows in Figure 2(a)).

In other words, the contaminant, here salt, was injected into one of the inlets, 3 m upstream of the cross junction shown as a black-filled box in Figure 2(b). The concentrations of salts were estimated using conductivity measurements (the concentration of salt in mg/l was estimated by the conductivity caused by salt in μS/cm multiplied by 0.65), 0.5 m upstream of the inlets (3 m away from the injection point) and at least 3 m downstream of the cross junction where the mixing occurred for the outlets.

The flow rate scenarios were implemented using flow control valves and flow meters, while the pressure was set with the help of the Berkeley pump and the pressure meters installed in all four legs of the cross junction. The plan of the experiment and the placement of facilities are illustrated in Figure 2(b). In each flow rate scenario (Table 1), two pressures (320 and 430 kPa) and two cross junctions with pipe diameters of 100 and 150 mm (Table 2) were tested. Since the tested pipes are made of PVC, it was assumed that their roughness was about 0.0015 mm (Dahl *et al.* 1990), which means that their relative

Table 2 | Pipe relative roughness and Re number of flow rates for cross junctions in laboratory experiments

Cross-junction pipe diameters (mm)	Pipe relative roughness	Re for flow rate 1.50 L/s	2.00 L/s	2.25 L/s	2.50 L/s	3.0 L/s
100 × 100 × 100 × 100	1.50×10^{-5}	19,098	25,464	28,647	31,830	38,197
150 × 150 × 150 × 150	1.00×10^{-5}	12,732	16,976	19,098	21,220	25,464

roughness was 1.5×10^{-5} and 1.0×10^{-5}, respectively, for the 100 and 150 mm pipes. Ten different salt concentrations were injected into the southern pipe in order to make sure that the results were replicable. Consequently, a total of 1,000 experiments were carried out (25 flow rate scenarios * 2 pressure values * 2 cross junctions * 10 salt concentration).

The dimensionless concentration of the injected salt in each outlet, obtained from the following equation, was chosen to represent the level of mixing:

$$C^* = \frac{C - C_w}{C_s - C_w} \tag{2}$$

where C^* is the dimensionless concentration of the injected salt in the studied outlet (north or east), C is the concentration of dissolved solids (ions) in the studied outlet, C_s is the concentration of dissolved solids (including the injected salt) in the southern inlet (ranging from 400 to 800 mg/L), and C_w is the concentration of dissolved solids in the tap water (ranging from 200 to 400 mg/L) coming from the western inlet. Finally, since 10 different quantities of salt were injected for each flow rate scenario, the dimensionless concentration of the injected salt for each scenario (Equation (3)) was obtained by taking the average of dimensionless concentrations for 10 quantities of injected salt:

$$\overline{C^*} = \frac{\sum_{i=1}^{10} C_i^*}{10} \tag{3}$$

where C_i^* is the dimensionless concentration in any outlet for the ith of 10 experiments carried out in each flow rate scenario. The absolute error propagation technique was used to estimate the uncertainty of the dimensionless concentration in any outlet. The uncertainty of the dimensionless concentration of injected salt in each outlet was estimated to be about ± 0.05; more details about the uncertainty calculations are provided in the Supplementary Materials.

To evaluate the accuracy of the six mixing models, the root mean square error (RMSE) was calculated between the C^* values computed by each model and the $\overline{C^*}$ values obtained from the experimental laboratory results. The equation for RMSE is presented in the following equation.

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{N} (C^*_{\text{model}} - \overline{C^*_{\text{laboratory experiments}}})^2}{N}} \tag{4}$$

In this equation, N is the number of cases tested by each IM model.

RESULTS AND DISCUSSIONS

Overall analysis

The RMSE of the six models is presented in Table 3. As can be seen from Table 3, the AZRED model has the lowest RMSE, while the models by Yu *et al.* (2014) and Shao *et al.* (2014) have the closest RMSE to each other, which are only slightly higher than the RMSE for AZRED. The performance of these three IM models is better because they are based on experimental results, while not relying on a scaling parameter that needs to be calibrated, as in EPANET-BAM, nor being limited to 12 scenarios, as for the model of Hernàndez Cervantes *et al.* (2021). Results in Table 3 also show that the analytical IM model by

Table 3 | RMSE of the different IM models over all tested flow rate scenarios

Model	RMSE
AZRED by Austin *et al.* (2008)	0.05
EPANET-BAM by Ho (2008)	0.18
Analytical IM model by Yu *et al.* (2014)	0.08
Analytical IM model by Shao *et al.* (2014)	0.07
Analytical IM model by Hernández Cervantes *et al.* (2021)	0.13
Analytical IM model by Hammoudi (2021)	0.39

Hammoudi (2021) has a higher RMSE value than the other models. The high RMSE of this model may be due to the fact that the authors only investigated the mixing with numerical simulations (without any laboratory experiment) and that the employed numerical model (Ansys CFX 19.1) was not calibrated. Therefore, this model was excluded from the following analyses.

For the five other IM models, Figure 3 shows the C^* values computed by each model and the $\overline{C^*}$ values obtained from the experimental laboratory results for all flow rate scenarios and for pipe diameters of $100 \times 100 \times 100 \times 100$ mm (or pipe relative roughness of 1.50×10^{-5}). Figure 4 shows the distribution of the absolute difference between the C^* values computed by each model and the $\overline{C^*}$ value obtained from the laboratory experimental results for all scenarios and both pipe diameters. In Figure 4, the quartiles of errors are shown by the three lines in each box. The lower and higher range limits of the bars are calculated based on the following equations:

$$\text{lower range limit} = Q_1 - 1.5 \times (Q_3 - Q_1) \tag{5}$$

$$\text{higher range limit} = Q_3 + 1.5 \times (Q_3 - Q_1) \tag{6}$$

where Q_1 is the first quartile and Q_3 is the third quartile.

Interpretation of the results

Figures 3 and 4 are quite revealing in several ways. First, the AZRED model and the model by Shao *et al.* (2014) provided results that are closer to the values observed during the laboratory experiments than the other three models. Second, the poorest results obtained with the EPANET-BAM model confirm that the scaling parameter needs to be calibrated. Indeed, for each flow rate scenario (or, in other words, for each inlet and outlet Re number ratio), the effect of diffusion on mixing changes, and, consequently, the value of the scaling parameter should be modified. Third, the IM model by Hernández Cervantes *et al.* (2021) shows the largest spread of errors; this could be explained by the fact that these authors limited the outputs of their model to only 12 flow rate scenarios.

Although the AZRED model and the model by Shao *et al.* (2014) showed the best global agreement with the experimental results, five outliers appear in Figure 4 (three for AZRED and two for the model by Shao *et al.* 2014). With the AZRED model, it was observed that in both 100 and 150 mm cross junctions, the outliers are associated with flow rate scenarios 1, 2, and 3 (see Table 1). In other words, when the inlet flow (or Re numbers) ratio was 0.50, and the outlet flow (or Re numbers) ratio was less than one, the AZRED model had the most elevated error among the studied flow rate scenarios. For the model by Shao *et al.* (2014), the outliers were for scenario 11 within the cross junctions with pipe diameters of 100 and 150 mm. Similarly to AZRED, the model by Shao *et al.* (2014) had its greatest error when the inlet flow (or Re numbers) ratio was 1.00, and the outlet flow (or Re numbers) ratio was around 0.50.

The dimensionless concentration values in Figure 3(a) that are higher than 1 are related to the model by Hernàndez Cervantes *et al.* (2021) and the laboratory experimental results of the current study. For those that are coming from the laboratory experimental results, this happened due to the uncertainties in the measurement of the results. However, all the laboratory experimental results, which are higher than one, are still lower than 1.05 (which is one plus the uncertainty of the laboratory experiments, 0.05).

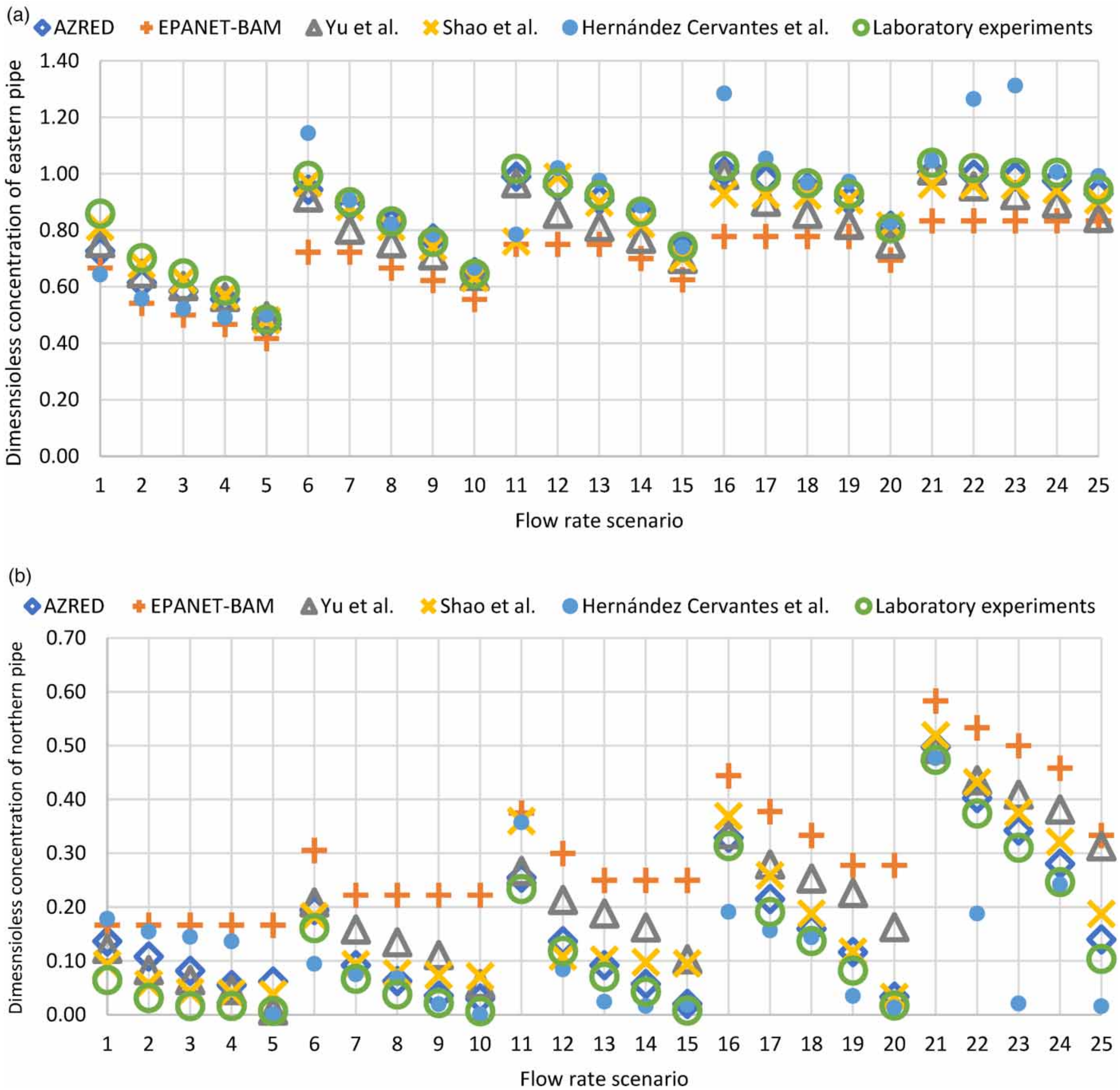


Figure 3 | Results of five IM models and laboratory experiments for the cross junction with pipe diameters of 100 mm in terms of the dimensionless concentration of the eastern pipe (a) and the dimensionless concentration of the northern pipe (b) within all flow rate scenarios.

For the Hernàndez Cervantes *et al.* (2021) model, the dimensionless concentration values in Figure 4(a) that are higher than 1 can be explained by the fact that the concentrations in the outlets of the studied scenario are assumed to be the same as the values of one of the 12 laboratory experiments by Hernàndez Cervantes *et al.* (2021), while the flows are not exactly the same.

Figure 3(a) shows that all the models except for the Hernàndez Cervantes *et al.* model underestimate the dimensionless concentration in the eastern pipe. According to the research by Orear *et al.* (2005), if the pipe diameters of two cross junctions are different, the degree of mixing in the smaller pipe diameter junction would be higher. In other words, the dimensionless concentrations in both outlets of that smaller pipe diameter junction would be closer to each other.

Here, one possible reason for this underestimation in eastern dimensionless concentrations could be the difference between the pipe diameter of cross junctions tested in the laboratory experiments (100 and 150 mm) and the pipe diameter of laboratory experiments carried out to develop each studied model (ranging between 25 and 50 mm).

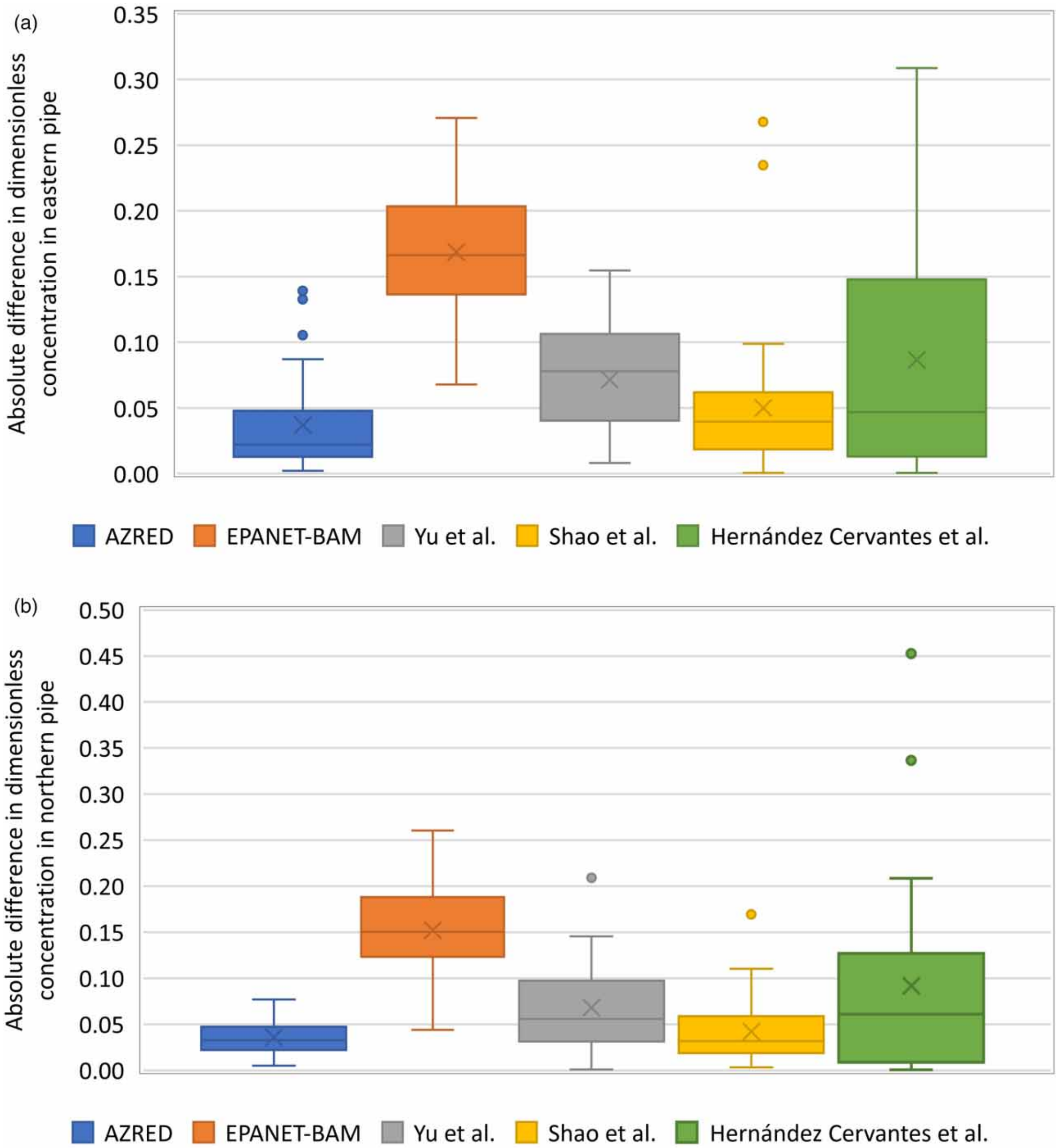


Figure 4 | Absolute difference between the experimental results and the results of five IM models under real-world conditions in terms of the dimensionless concentration of the eastern pipe (a) and the dimensionless concentration of the northern pipe (b).

CONCLUSIONS

Experimental investigations were carried out to evaluate six existing IM models and to make recommendations for using those models under real-world conditions. Based on this study and investigations, it can be concluded that:

- Among the existing IM models, the AZRED model and the model by Shao *et al.* (2014) showed the least differences in experimental results, with the AZRED model having the least RMSE.
- The AZRED model could not reproduce the experimental results when the inlet Re number ratio (i.e., the ratio of the Re for the west inlet to the south inlet) was 0.50, and the outlet Re number ratio (i.e., the ratio of the Re for the east outlet to the north outlet) was less than one, while the IM model by Shao *et al.* (2014) gave results that were far different from the experimental results when the inlet Re number ratio was around 1.00, and the outlet Re number ratio was 0.50.

This study sheds light on the performance of existing IM models for cross junctions with the same pipe diameter in all four legs during conditions encountered within real WDNs. However, the findings summarized above are subject to some limitations. First, the studied flow rate scenarios were limited to 1.50 and 3.00 L/s. Second, the mixing in cross junctions with pipe diameters greater than 150 mm was not investigated in this study. Notwithstanding these limitations, the study suggests that the AZRED model has the highest potential to be used for projecting the mixing in cross junctions when the inlet Re number ratio (i.e., the ratio of the Re for the west inlet to the south inlet) is between 0.50 and 2.00 and the outlet Re number ratio (i.e., the ratio of the Re for the east outlet to the north outlet) is between 1.00 and 2.00.

There are also many unanswered questions that require further investigation. For instance, additional research could evaluate the performance of the IM models under higher inlet or outlet Re number ratios. More broadly, similar research is also needed to evaluate the performance of the existing IM models for double-tee junctions and for junctions with unequal pipe diameters. The results of this study, along with the mentioned future areas of research, can help to improve the IM models and increase their use for real-world WDN applications such as managing disinfectant residuals, water quality sensor placement, and contaminant source identification. Even so, three of the tested models (AZRED and the ones by Yu *et al.* (2014) and Shao *et al.* (2014)) were found to well reproduce the mixing conditions in cross junctions. This type of junction is frequently encountered in both community scale and transmission networks in North America, which means that the three models mentioned above could help improve the design of booster stations (location and injection schedule) as well as the identification of potential contaminant sources in these networks. Applications on networks where double-tee and wye junctions are dominant should be based on models tested for these conditions in future research work.

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DATA AVAILABILITY STATEMENT

All relevant data are included in the paper or its Supplementary Information.

CONFLICT OF INTEREST

The authors declare there is no conflict.

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