

Quintessentially Flat Scalar Potentials

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ABSTRACT: Both inflationary and quintessence cosmologies require scalar fields which roll very slowly over cosmological time scales, and so typically demand extremely flat potentials. Sufficiently flat potentials are notoriously difficult to obtain from realistic theories of microscopic physics, and this poses a naturalness problem for both types of cosmologies. We propose a brane-world-based microscopic mechanism for generating scalar potentials which can naturally be flat enough for both types of cosmological applications. The scalars of interest are higher-dimensional bulk pseudo-Goldstone bosons whose scale of symmetry breaking is exponentially suppressed in the higher-dimensional theory by the separation between various branes. The light scalars appear in the effective 4D theory as pseudo-Goldstone bosons. Since naturalness problems are more severe for quintessence models, motivated by our construction we explore in more detail the possibilities for using pseudo-Goldstone bosons to build quintessence models. Depending on how the cosmological constant problem is solved, these models typically imply the universe is now entering a matter-dominated oscillatory phase for which the equation of state parameter $w = p/\rho$ oscillates between $w = 1$ and $w = -1$.

KEYWORDS: brane world, quintessence, inflation.

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1. Introduction

Perhaps the most interesting consequence of the recent spate of cosmological measurements is the accumulation of evidence suggesting the Universe has passed through no less than *two* independent periods of acceleration during that part of its history to which we have observational access. The first of these periods is the early inflationary period [1], whose simplest predictions for the temperature fluctuations of the cosmic microwave background (CMB) radiation appear to describe very successfully what is seen [2]. The big surprise of the past decade is the discovery that the present epoch also appears to be a period of incipient inflation, as indicated by both CMB measurements [3] and supernova surveys [4].

Both of these epochs can be described by the slow roll of a scalar field since they are both defined by the condition that the universal expansion accelerates, and this in turn requires the dominant contribution to the energy density, ρ , to have sufficiently negative pressure: $p < -\rho/3$.¹ If the dominant energy is due to a rolling homogeneous scalar field, then its pressure-to-energy ratio is related to the fraction, $r = K/V$, of the scalar field's kinetic and potential energies according to $p/\rho = (r - 1)/(r + 1)$. This shows that acceleration is possible only if the scalar field is presently potential-energy dominated: $K \lesssim V$. For inflation the corresponding energy density is typically chosen to be $\rho \approx V \lesssim (10^{15} \text{ GeV})^4$, while applications to the present epoch — which we generically refer to as ‘quintessence’ models [5] — instead require $\rho \approx V \sim (10^{-3} \text{ eV})^4$.

Both of these applications of slow-roll scalar fields run into difficulties because of the flatness of the potential which they require. The problem is the notorious difficulty in obtaining very flat potentials from realistic theories of microscopic physics [6]. Our purpose in this paper is to propose a new mechanism for obtaining extremely flat potentials from within a brane-world picture [7, 8]. In the model we propose, the scalar of interest is a pseudo-Goldstone boson [9, 10] for an approximate symmetry (more about which below) which is explicitly broken, but whose breaking requires the presence of more than one brane as well as of a massive field living in the bulk between the branes. This combination ensures that the low-energy effective potential is suppressed by the amplitude, \mathcal{A} , for the massive particle to propagate from one brane to another, which is exponentially small in the inter-brane separation, a , in units of the massive-particle Compton wavelength, M^{-1} : $\mathcal{A} \sim \exp(-Ma)$.

The paper is organized in the following way. In the next section we outline how flat the scalar potentials must be for cosmological applications to inflation and quintessence, and summarize the naturalness problems which one encounters trying to obtain potentials this flat. This section also very briefly reviews what it means for the scalar be a pseudo-Goldstone boson (pGB), and why this can help with the naturalness issues. Since this section is not particularly new (see refs. [11, 12, 13] for applications of pseudo-Goldstone bosons to cosmology), the professionals will want to skip directly to the next section, §3, where we describe our brane-world model, and show why it can give such flat potentials. At low energies the scalar model we produce is a pseudo-Goldstone boson, and so motivated by this in §4 we build an explicit quintessence model in order to show a detailed example of a successful cosmology using pseudo-Goldstone bosons, updating the earlier models of refs. [11, 12]. In this section we also re-examine the viability of these models in the light of recent WMAP measurements, and identify potentially-observable differences between this kind of cosmology and other proposals. Finally, our conclusions are summarized in section §5.

¹We assume here the universe to be spatially flat, $k = 0$.

2. Slow-Rolling Scalars and Cosmology

In this section we have two goals. First, we review the general constraints which cosmological applications require of slowly-rolling scalar fields, to see what kinds of hierarchies of scale a successful cosmology requires of an underlying theory. Then we examine pseudo-Goldstone bosons in particular, and ask how large the corresponding hierarchies are related to the corresponding energy scales for the various types of symmetry breaking. In this second section we identify two cases, which differ in whether or not the largest symmetry-breaking effects arise in the scalar kinetic energies or in the scalar potential.

Since the arguments in this section are relatively standard, experts should feel free to skip directly to section §3.

2.1 Constraints Required by Cosmologically Slow Rolls

A problem with applications of rolling scalar fields to both inflation and to quintessence cosmologies arises because they each require an inordinately flat potential. We here summarize these constraints subject to very mild assumptions.

In general, a scalar roll is only slow enough to neglect its kinetic energy if the slow-roll parameters [14] $\epsilon = \frac{1}{2}(M_p V'/V)^2$ and $\eta = M_p^2 V''/V$ are much smaller than unity. (Here $M_p \approx 10^{18}$ GeV is the 4D Planck mass and the prime denotes differentiation with respect to the canonically-normalized scalar fields.) To see what this requires suppose the scalar action has the generic form

$$-\frac{\mathcal{L}}{\sqrt{-g}} = \frac{f^2}{2} (\partial\varphi)^2 + \mu^4 v(\varphi), \quad (2.1)$$

where $0 \leq \varphi(x) \leq 2\pi$ is a dimensionless field and f and μ are constants having dimensions of mass (in units for which $\hbar = c = 1$). If we suppose that $v(\varphi)$ and all of its derivatives are $O(1)$, then $\epsilon \sim \eta \sim M_p^2/f^2$ which shows that we must require $f \gg M_p$, in which case the scalar mass, $m \sim \mu^2/f$, must be much smaller than the Hubble scale, $H = (\rho/3M_p^2)^{1/2} \sim \mu^2/M_p$.

Inflation

In order for such a rolling scalar in an early inflationary period to properly describe the amplitude of CMB temperature fluctuations requires the combination $\delta^2 = (1/150\pi^2)(V/M_p^4\epsilon)$ must satisfy $\delta \approx 2 \times 10^{-5}$. Using the conditions $\epsilon \sim M_p^2/f^2$ and $H \sim \mu^2/M_p$ just described, implies $\mu/M_p \sim 0.03 \sqrt{M_p/f}$. Together with the observational constraint [2] $\epsilon < 0.03$, we find the requirement $M_p/f \lesssim 0.2$ and $\mu/M_p \lesssim 0.006$.

Quintessence

On the other hand, the situation is even worse if the scalar is to describe today's Universal acceleration, since such a scalar must satisfy $\mu \sim 10^{-3}$ eV. This, with the slow-roll condition $f \gtrsim M_p$, leads to $\mu/f \lesssim \mu/M_p \sim 10^{-30}$ and the incredibly small scalar mass $m \sim \mu^2/f \lesssim \mu^2/M_p \sim 10^{-33}$ eV.

2.2 Naturalness Issues

It is notoriously difficult to get very flat scalar potentials from realistic microscopic physics without fine-tuning, and this difficulty comes in two parts. First one must ask: *How do the small ratios μ/f and μ/M_p arise within the microscopic theory as a combination of microscopic parameters?* Given that such a small ratio is predicted by the microscopic physics, one must then ask: *How does it remain small as one integrates out all the physics between these microscopic scales and the cosmological scales at which it is measured?*

Of these, the second problem is the more serious, the more so the lower μ is required to be. It is a problem because a particle of mass M , which interacts with the scalar with order-unity couplings, typically shifts μ by an amount $\delta\mu \propto M$ when it is integrated out, which can be unacceptably large if $M \gtrsim \mu$. There are two symmetries which are known to be able to help with this problem, in that they can ensure that particles of mass M do not produce corrections as large as $\delta\mu \sim M$. The two symmetries are: (1) supersymmetry, for which bose-fermi cancellations ensure $\delta\mu \lesssim M_s$, where M_s is the typical mass splitting within a supermultiplet; (2) Goldstone symmetries, for which the scalar transforms inhomogeneously according to $\delta\varphi = \epsilon[1 + F(\phi)]$, where ϵ is the transformation parameter and the potentially nonlinear function F satisfies $F(0) = 0$.

This second type of symmetry arises only if the scalar in question is a Goldstone boson for a spontaneously broken global symmetry, and it ensures $v(\varphi)$ must be completely independent of φ . $v(\varphi)$ can be nontrivial if the global symmetry is only approximate, in which case corrections to μ are systematically suppressed by whatever small symmetry-breaking parameter makes the symmetry a good approximation. In this case the scalar φ is known as a pseudo-Goldstone boson [9, 10].

2.3 Pseudo-Goldstone Bosons

The scales μ and f are related to the scales of symmetry breaking in the underlying microscopic theory. Once set there they naturally remain small as successive scales are integrated out to obtain an effective theory at very low energies. These low-energy corrections remain small precisely because the scalar φ is a pseudo-Goldstone boson, and so corrections to μ are protected by the nonlinearly-realized G symmetry.

A lower limit to the amount of this suppression can be inferred completely within the low-energy theory by power-counting within it the size of loop-generated

symmetry-breaking corrections [10]. To this end consider a system of N scalars, φ^a , where we choose to rescale the spacetime metric to go to the Einstein Frame, for which the graviton kinetic term takes the canonical Einstein-Hilbert form. The scalar part of the lagrangian density which involves the fewest derivatives may always be written

$$\frac{\mathcal{L}_s}{\sqrt{-g}} = -V(\varphi) - \frac{1}{2} G_{ab}(\varphi) g^{\mu\nu} \partial_\mu \varphi^a \partial_\nu \varphi^b. \quad (2.2)$$

The symmetric tensor G_{ab} may be interpreted as a metric on the scalar-field ‘target’ space. For a single scalar field G_{ab} may be set to unity by an appropriate field redefinition, and so in this case it is the scalar potential, $V(\varphi)$, which determines all of the physics. A similar choice, $G_{ab} = \delta_{ab}$ is *not* possible if $N \geq 2$, however, unless the target space happens to be flat. In order to avoid missing physics associated with G_{ab} we consider models below involving two or more pseudo-Goldstone scalars.

When the scalars φ^a are Goldstone bosons the functions V and G_{ab} are strongly restricted by symmetry conditions. These imply V must be a constant, and for the symmetry-breaking pattern $G \rightarrow H$, G_{ab} must be a metric on the coset space G/H whose isometries include the symmetry group G . This usually determines G_{ab} up to a few constants, and often completely determines it up to overall normalization and field redefinitions [15, 10].

For example, for the symmetry-breaking pattern $SO(3) \rightarrow SO(2)$ there are then two Goldstone bosons, $(\varphi^1, \varphi^2) = (\theta, \phi)$, which parameterize the coset space $SO(3)/SO(2)$, which in this case is the two-sphere, S_2 . Here we use standard spherical-polar coordinates, $0 \leq \theta < \pi$ and $0 \leq \phi \leq 2\pi$, on S_2 . The $SO(3)$ transformations amount to the rotations of this sphere about its centre, if S_2 is embedded into Euclidean three-dimensional space. The condition that the action be invariant under $SO(3)$ transformations then requires $V(\theta, \phi)$ to be constant, and $G_{ab}(\theta, \varphi)$ to be the standard rotationally-invariant – ‘round’ – metric on the 2-sphere:

$$G_{ab} \partial_\mu \varphi^a \partial^\mu \varphi^b = f^2 \left(\partial_\mu \theta \partial^\mu \theta + \sin^2 \theta \partial_\mu \phi \partial^\mu \phi \right). \quad (2.3)$$

f is a dimensionful constant whose size indicates the scale of spontaneous symmetry breaking.

Our interest here is in pseudo-Goldstone bosons, for which the global symmetry G is only approximate in the sense that the effective energy scale, μ , associated with the explicit breaking of the symmetry is much smaller than the scale, f , of its spontaneous breaking. (We have already seen that this effective scale need not be simply related to the microscopic scales of the microscopic theory.) In this case V need no longer be independent of φ^a and G_{ab} need not be a G -invariant metric, although deviations from these limits should be small if the scale, μ , is much smaller than the scale, f , of spontaneous symmetry breaking.

In the limit $f \gg \mu$ on dimensional grounds we expect the generic corrections to V to be of order μ^4 and corrections to G_{ab} to be of order μ^2/f^2 . If the asymmetric

terms are initially this size, they automatically remain so after being renormalized by quantum corrections within the low-energy theory. As is briefly summarized in the appendix, these orders of magnitude can differ in supersymmetric theories, if μ is larger than the supersymmetry breaking scale.

For instance, for the $SO(3)/SO(2)$ example, suppose the $SO(3)$ symmetry is explicitly broken but the $SO(2)$ symmetry associated with shifting ϕ is not. Then examples of the kinds of new terms one might expect at low energies might be

$$V(\varphi) = a + \sum_{n \geq 1} b_n \cos(n\theta) \quad (2.4)$$

$$G_{ab} d\varphi^a d\varphi^b = f^2 [d\theta^2 + G(\theta) d\phi^2 + \dots] ,$$

where
$$G(\theta) = \sin^2 \theta + \sum_{n \geq 2} c_n \sin^2(n\theta) , \quad (2.5)$$

where the sums run over integer values. The above dimension counting then argues that while a need not be suppressed by μ , we expect $b_n \lesssim \mu^4$ and $c_n \lesssim \mu^2/f^2$.

In applications it is usual to neglect the corrections to G_{ab} and keep only the scalar potential which is induced by explicit symmetry breaking. This is usually justified because the symmetry-breaking potential always dominates at low energies because there is no symmetry-invariant potential with which to compete. For instance, in cosmological applications Hubble damping inevitably slows the scalar motion, making the potential a more and more important influence on the scalar roll. The same is not true for the corrections to the target-space metric, G_{ab} , since these are always at most of order μ^2/f^2 relative to the G -invariant metric of the symmetry limit.

Global Symmetries and Gravity

The power-counting statements made above assume that quantum corrections respect the theory's underlying G invariance. Unfortunately, there is an important kind of quantum correction which may not do so for any global symmetry, and this represents a potential obstacle to using a pseudo-Goldstone symmetry to keep the corrections to μ small.

The problem comes from gravitational quantum corrections, which are believed not to respect global symmetries, for instance due to the virtual appearance and disappearance of black holes (which the ‘no-hair’ theorems ensure cannot carry global-symmetry charges). Estimates [16] of the amount of symmetry breaking which this induces in a low-energy 4D effective theory predict that the symmetry-breaking interactions are suppressed by powers of f^2/M_p^2 . This can represent an important renormalization to μ precisely for the case of cosmologically slowly-rolling fields, for which we've seen $f \gtrsim M_p$.

One must keep in mind that these estimates of non-perturbative quantum-gravity effects carry the caveat that they assume many things about the properties of quan-

tum gravity at high energies, and so may not properly capture how things work once this high-energy physics is better understood. In particular, as pointed out in ref. [16], these symmetry-breaking estimates can change dramatically if the effective theory becomes higher dimensional at scales $M_c \ll M_p$, as is the case in the brane-world models we describe below. Of course, how small a quantum gravity correction may be tolerated depends very much on how flat a scalar potential is desired, making all of these issues much more pressing for present-epoch quintessence models than they are for inflation.

In what follows we proceed under the assumption that this, or a similar mechanism, ensures that high-energy quantum-gravity effects do not destroy the effectiveness of the pseudo-Goldstone boson mechanism in protecting the flatness of the scalar potential to the accuracy required for cosmology.

3. Flat Scalar Potentials from the Brane World

Although pseudo-Goldstone bosons can have naturally flat potentials if $\mu \ll f$, they do not in themselves explain why μ should be so small. An understanding of this must come from a more microscopic theory. In this section we describe a brane-world model within which such small scales can arise for pseudo-Goldstone boson potentials. Brane models are natural to examine from this point of view, because in many situations they have given new insights on how small quantities can arise in low-energy low-energy physics [17].

The idea behind our construction is to make a model having a $G = U_A(1) \times U_B(1)$ global symmetry which is spontaneously broken by the vev , v , of a bulk scalar field Φ . The symmetry is also broken explicitly by the couplings of a second bulk scalar field Ψ (having a large mass M) to various brane fields χ_i . In particular, the model is designed so that there is more than one brane (say two of them) and only the $U_A(1)$ symmetry is broken by the Ψ couplings to the first brane, and only the $U_B(1)$ symmetry is broken by the Ψ couplings to the second brane which is displaced a distance a away from the first brane. Once this is arranged, functional integration over Ψ and the brane modes generates a nontrivial scalar potential for the would-be goldstone mode in Φ , which is suppressed by the amplitude $\mathcal{A} \propto \exp(-Ma)$ for the field Ψ to propagate from one brane to the other. The logic of this construction is reminiscent of brane-based supersymmetry breaking mechanisms, for which each brane preserves some supersymmetries but where all supersymmetries are broken by at least one brane [18].

3.1 The Higher-Dimensional Toy Model

Consider, then, a model containing the complex scalar bulk fields Φ and Ψ , and complex brane fields $\chi_i, i = 1, 2$, whose action is $S = S_B + S_{b1} + S_{b2}$, with $(4 + n)$ -

dimensional bulk action

$$S_B = - \int d^4x d^n y \left[(\partial\Psi)^*(\partial\Psi) + (\partial\Phi)^*(\partial\Phi) + V(\Phi, \Psi) \right]$$

where $V(\Phi, \Psi) = M^2 \Psi^* \Psi + \frac{\lambda}{2} (\Phi^* \Phi - v^2)^2$, (3.1)

and 4-dimensional brane actions

$$S_{b1} = - \int_{y=y_1} d^4x \left[(\partial\chi_1)^*(\partial\chi_1) + m_1^2 \chi_1^* \chi_1 + \frac{1}{2} [(g_1\Phi + h_1\Psi) \chi_1^2 + \text{c.c.}] \right]$$

$$S_{b2} = - \int_{y=y_2} d^4x \left[(\partial\chi_2)^*(\partial\chi_2) + m_2^2 \chi_2^* \chi_2 + \frac{1}{2} [(g_2\Phi + h_2\Psi^*) \chi_2^2 + \text{c.c.}] \right]. \quad (3.2)$$

We assume all of the couplings, λ , g_i and h_i to be real and nonzero, but sufficiently small to permit a perturbative analysis of the model. We imagine the branes to be parallel 3-branes which are situated at the points $y = y_i$ within the n compact transverse dimensions. We take the size of all of these dimensions to be of the same order, r , making the compactification scale (Kaluza-Klein masses) of order $M_c \sim 1/r$.

By construction, the model enjoys a global $G = U(1) \times \tilde{U}(1)$ symmetry under which each of the bulk scalars rotate independently: $\Phi \rightarrow e^{i\omega} \Phi$ and $\Psi \rightarrow e^{i\tilde{\omega}} \Psi$. Other perturbative couplings could also be permitted in the bulk scalar potential without substantially changing our conclusions, provided they also respect this symmetry.

The brane couplings, on the other hand, each explicitly break this symmetry down to a single $U(1)$. The bulk-brane couplings at brane 1 preserve the subgroup $U_A(1)$ under which $\Phi \rightarrow e^{i\omega_A} \Phi$, $\Psi \rightarrow e^{i\omega_A} \Psi$ and $\chi_1 \rightarrow e^{-i\omega_A/2} \chi_1$. Similarly the couplings on brane 2 only preserve the subgroup $U_B(1)$ under which $\Phi \rightarrow e^{i\omega_B} \Phi$, $\Psi \rightarrow e^{-i\omega_B} \Psi$ and $\chi_2 \rightarrow e^{-i\omega_B/2} \chi_2$. Taken together, both branes completely break the symmetry group $U(1) \times \tilde{U}(1)$. Notice also that it is only the field Ψ which transforms differently under $U_A(1)$ and $U_B(1)$, and so both the brane and bulk actions would preserve one of the $U(1)$'s if the field Ψ were everywhere set to zero.

The spectrum of this model is easy to understand in the limit $h_i \rightarrow 0$, in which case the brane couplings also preserve the G symmetry. Then the nonzero expectation value $\langle \Phi \rangle = v$ spontaneously breaks the $U(1)$ symmetry, while leaving the $\tilde{U}(1)$ unbroken. In the absence of the branes, therefore, the bulk theory would consist of a mass- M complex field Ψ plus the two real mass eigenstates coming from Φ . One of the Φ mass eigenstates would in this case be a massless Goldstone boson, $\varphi = \arg \Phi$, and the other would have a mass of order $\sqrt{\lambda} v$. We now compute how nonzero h_i couplings on the brane change these conclusions.

3.2 The Effective 4D Theory

Since our interest is in the would-be Goldstone boson, we focus on the low-energy theory below the compactification scale, by integrating out all of the massive fields on

the branes and in the bulk. In particular, we concentrate on the effective scalar potential for the would-be Goldstone mode, φ , in this low-energy theory. We therefore look for those terms in the scalar potential which involve the phase of Φ , neglecting also the Kaluza-Klein tower of compactification modes for this field.

Integrating out the Brane Modes

We start by integrating out the brane scalars, with the bulk fields held fixed. The leading contribution arises at one loop, leading to a scalar-potential contribution to the effective $(4 + n)$ -dimensional bulk theory of the form

$$\delta\mathcal{L}_B = \sum_{i=1}^2 \Delta V_{\text{eff},i} \delta^n(y - y_i), \quad (3.3)$$

where

$$\Delta V_{\text{eff},i} = \frac{1}{64\pi^2} \text{Tr} \left[\mathcal{M}_i^4 \log \left(\frac{\mathcal{M}_i^2}{\mu^2} \right) \right], \quad (3.4)$$

and the trace is over the two components of the χ_i mass matrix

$$\mathcal{M}_i^2 = \begin{pmatrix} m_i^2 & \xi_i \\ \xi_i^* & m_i^2 \end{pmatrix}. \quad (3.5)$$

Here μ is an arbitrary renormalization scale, $\xi_1 = g_1 \Phi + h_1 \Psi$ and $\xi_2 = g_2 \Phi + h_2 \Psi^*$. Evaluating the trace we find

$$\begin{aligned} \Delta V_{\text{eff},i} &= \frac{1}{64\pi^2} \left[(m_i^2 + |\xi_i|)^2 \log \left(\frac{m_i^2 + |\xi_i|}{\mu^2} \right) + (m_i^2 - |\xi_i|)^2 \log \left(\frac{m_i^2 - |\xi_i|}{\mu^2} \right) \right] \\ &\approx \frac{1}{32\pi^2} \left[m_i^4 \log \left(\frac{m_i^2}{\mu^2} \right) + |\xi_i|^2 \left(\frac{3}{2} + \log \left(\frac{m_i^2}{\mu^2} \right) \right) + \dots \right], \end{aligned} \quad (3.6)$$

where we assume m_i to be large enough to ensure that $m_i^2 > |\xi_i|$ for $\Phi \sim v$, and the second, approximate, equality applies for $|\xi_i| \ll m_i^2$.

Integrating out Ψ

We next integrate out the massive bulk field, Ψ , with Φ temporarily held fixed. The leading result in this case arises at tree level, corresponding to the elimination of Ψ from the classical action (supplemented by the effective brane-induced interaction, eq. (3.3)), using its classical field equation

$$\left(-\square + M^2 \right) \Psi_c = - \sum_{i=1}^2 \delta^n(y - y_i) \frac{\partial \Delta V_{\text{eff},i}}{\partial \Psi^*}, \quad (3.7)$$

which, using eq. (3.6), gives the approximate expression

$$\begin{aligned} \left(-\square + M^2 \right) \Psi_c &\approx - \delta^n(y - y_1) \frac{h_1 \xi_1}{32\pi^2} \left[\frac{3}{2} + \log \left(\frac{m_1^2}{\mu^2} \right) \right] \\ &\quad - \delta^n(y - y_2) \frac{h_2 \xi_2^*}{32\pi^2} \left[\frac{3}{2} + \log \left(\frac{m_2^2}{\mu^2} \right) \right]. \end{aligned} \quad (3.8)$$

Working to leading order in powers of h_i allows us to write $\xi_i \approx g_i \Phi$ in this last equation, allowing its solution to be written

$$\Psi_c(y) \approx -\frac{1}{32\pi^2} \left\{ h_1 g_1 \Phi(y_1) \left[\frac{3}{2} + \log \left(\frac{m_1^2}{\mu^2} \right) \right] G(y, y_1) + h_2 g_2 \Phi^*(y_2) \left[\frac{3}{2} + \log \left(\frac{m_2^2}{\mu^2} \right) \right] G(y, y_2) \right\}. \quad (3.9)$$

Here $G(y, y')$ is the solution to $(-\square + M^2)G(y, y') = \delta^n(y, y') - 1/\Omega_n$, where Ω_n denotes the volume of the n extra dimensions. $G(y, y')$ is given explicitly by the mode sum

$$G(y, y') = \sum'_\ell \frac{u_\ell(y) u_\ell^*(y')}{\lambda_\ell}, \quad (3.10)$$

in terms of the eigenvalues and eigenfunctions satisfying $(-\square + M^2)u_\ell(y) = \lambda_\ell u_\ell(y)$. The prime on the sum indicates the omission of any zero modes, for which $\lambda_\ell = 0$.

Substitution into the classical action, eqs. (3.1) and (3.2), then gives an action of the form $S_{\text{eff}}[\Phi] = S_{\text{inv}}[\Phi] + \Delta S[\Phi]$, where $S_{\text{inv}}[\Phi]$ is invariant with respect to $\Phi \rightarrow e^{i\omega} \Phi$, and

$$\Delta S[\Phi] \approx -k \int d^4x \left[g_1 g_2 h_1 h_2 \Phi(x, y_1) \Phi(x, y_2) G(y_1, y_2) + \text{c.c.} \right] + \dots. \quad (3.11)$$

In this last expression the constant k is given explicitly by

$$k = \left(\frac{1}{32\pi^2} \right)^2 \left[\frac{3}{2} + \log \left(\frac{m_1^2}{\mu^2} \right) \right] \left[\frac{3}{2} + \log \left(\frac{m_2^2}{\mu^2} \right) \right]. \quad (3.12)$$

Integrating out the Φ Kaluza-Klein Modes

The final step is to integrate out the massive Kaluza-Klein modes for Φ to obtain the effective four-dimensional action. Since the Kaluza-Klein zero mode for Φ is independent of the extra-dimensional coordinates y , to leading order this corresponds to simply truncating the action using $\Phi(x, y) \rightarrow \Phi(x)$.

Using the information that the invariant part of the potential is minimized for $\Phi(x) = v_R e^{i\varphi(x)} \neq 0$, where v_R is an appropriately renormalized parameter which differs from v because of the changes to the invariant part of the potential (which we do not follow here in detail), we obtain in this way the following effective action for the would-be Goldstone mode, φ :

$$S_{\text{eff}}[\varphi] = - \int d^4x \left[f^2 (\partial\varphi)^2 + V(\varphi) \right], \quad (3.13)$$

where $f^2 = v_R^2 \Omega_n$ with Ω_n as before denoting the volume of the internal dimensions. The low-energy scalar potential is given within the above approximations by $V \approx$

$\mu^4 \cos(2\varphi) + \dots$ (up to an additive constant, V_0 , which we may absorb into the renormalization of the cosmological constant). The constant μ is given approximately by

$$\mu^4 \approx 2 k g_1 g_2 h_1 h_2 v_R^2 G(y_1, y_2). \quad (3.14)$$

Eqs. (3.13) and (3.14) are the main results of this section.

3.3 Phenomenological Choices for the Scales

We see that the higher-dimensional model implies an effective 4D action for φ of the generic form of eq. (2.1), with the constants f and μ given in terms of more microscopic parameters. Given an internal space for which $\Omega_n \sim r^n$ and a brane separation a , we therefore find the order of magnitude results

$$f \sim v r^{n/2}, \quad (3.15)$$

and

$$\mu \sim (g_1 g_2 h_1 h_2)^{1/4} \sqrt{v} \left[\frac{1}{M} \left(\frac{M}{a} \right)^{(n-1)/2} e^{-Ma} \right]^{1/4}, \quad (3.16)$$

where we take v_R and v to be the same order of magnitude.

For comparison the 4D Planck mass is given by $M_p \sim M_g (M_g r)^{n/2}$, where M_g is the higher-dimensional gravitational scale. Eq. (3.16) uses the asymptotic form for the Green's function in the limit $Ma \gg 1$: $G(a) \sim M^{-1} (M/a)^{(n-1)/2} \exp(-Ma)$.

The exponential dependence of the heavy-field Green's function is what allows the scale μ to be naturally much smaller than f and M_p . For example, consider the simplest instance where we assume r is much larger than all other fundamental length scales, which we choose to all be of order M_g . Taking then $a \sim r \gg 1/M_g$, we therefore suppose the higher-dimensional theory to involve only a single scale, M_g , and so take $g_i \sim \hat{g}_i M_g^{1-n/2}$, $h_i \sim \hat{h}_i M_g^{1-n/2}$, $M \sim M_g$ and $v \sim M_g^{1+n/2}$. This leads to

$$f \sim M_p \sim M_g (M_g r)^{n/2} \quad \text{and} \quad \mu \sim \frac{(\hat{g}_1 \hat{g}_2 \hat{h}_1 \hat{h}_2)^{1/4}}{(M_g r)^{(n-1)/8}} M_g \exp\left(-\frac{M_g r}{4}\right). \quad (3.17)$$

This expression shows that the most natural choice for the higher-dimensional scales implies a large decay constant $f \sim M_p$, but with the ratio μ/f exponentially small given even only a moderately large value for $M_g r \gg 1$.

The exponential dependence on $M_g a$ allows the resulting scale μ to easily be small enough even for present-epoch applications. For instance taking $\hat{g}_i \sim \hat{h}_i \sim 1$ and $M_g r$ to be only slightly larger than the minimum size required to solve the electroweak hierarchy problem [8], $M_g r \sim 200$, $n = 6$ and $M_g \sim 10^{11}$ GeV, we have $\mu \sim 10^{-3}$ eV.

Applications to inflation are also possible provided it can be ensured that $f \gg M_p$. For instance this might be arranged in one of the above scenarios if $M_\Phi \sim$

$\sqrt{\lambda}v \sim M_g$, but with $\lambda \ll M_g^{-n/2}$. In this case the requirement $\mu \sim 10^{-4} M_p$ requires a smaller microscopic hierarchy. For instance if $n = 6$ then $M_g \sim 10^{15}$ GeV and $M_g r \sim 8$ does the job.

4. Pseudo-Goldstone Boson Cosmologies

Given the extremely shallow potentials which are possible with this mechanism, we next re-examine the cosmology of pseudo-Goldstone boson models in more detail. Our purpose in so doing is to reconsider more quantitatively the cosmological viability of these models in the light of present observations.

We first describe in general the cosmological rolling of several scalar fields in four dimensions, and then return to the specific cases where the scalars are pseudo-Goldstone bosons. Our purpose is to define our notation, and to highlight the features of generic pGB-based Quintessence models so these may be contrasted with what obtains for the usual axion-based models [11, 12].

4.1 General Multi-scalar Equations

The equations of motion which are obtained by varying the sum of the Einstein-Hilbert and the scalar action of eq. (2.2) produce the following equations of motion:

$$\begin{aligned} R_{\mu\nu} + \kappa^2 \left[G_{ab} \partial_\mu \varphi^a \partial_\nu \varphi^b + V(\varphi) g_{\mu\nu} \right] &= 0 \\ g^{\mu\nu} D_\mu \partial_\nu \varphi^a - G^{ab} V_{,b} &= 0, \end{aligned} \quad (4.1)$$

where $V_{,a} = \partial V / \partial \varphi^a$ and we adopt Weinberg's curvature conventions [19]. The spacetime and target-space covariant derivative, D_μ , for the scalar field which appears in eq. (4.1) is defined by:

$$\begin{aligned} D_\mu \partial_\nu \varphi^a &= \nabla_\mu \partial_\nu \varphi^a + \Gamma_{bc}^a(\varphi) \partial_\mu \varphi^b \partial_\nu \varphi^c \\ &= \partial_\mu \partial_\nu \varphi^a - \gamma_{\mu\nu}^\lambda \partial_\lambda \varphi^a + \Gamma_{bc}^a \partial_\mu \varphi^b \partial_\nu \varphi^c. \end{aligned} \quad (4.2)$$

$\gamma_{\nu\lambda}^\mu(x)$ is the usual Christoffel symbol constructed from the spacetime metric $g_{\mu\nu}(x)$ and $\Gamma_{bc}^a(\varphi)$ is the Christoffel symbol built from the target-space metric $G_{ab}(\varphi)$.

For cosmological applications we restrict these equations to a homogeneous but time-dependent field configuration and a Friedmann-Robertson-Walker (FRW) spacetime: $\varphi^a = \varphi^a(t)$, and

$$g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a^2(t) \gamma_{mn}(y) dy^m dy^n, \quad (4.3)$$

where γ_{mn} is the usual homogeneous metric on the surfaces of constant t , parameterized by $k = 0, \pm 1$. With these choices the equations of motion reduce to:

$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = \frac{\rho}{3M_p^2} - \frac{k}{a^2}$$

$$\begin{aligned} \frac{d}{da}(\rho a^3) &= -3p a^2 \\ \frac{D\dot{\varphi}^a}{dt} + 3H\dot{\varphi}^a + G^{ab} \frac{\partial V}{\partial \varphi^b} &= 0, \end{aligned} \quad (4.4)$$

where

$$\begin{aligned} \rho &= \frac{1}{2} G_{ab} \dot{\varphi}^a \dot{\varphi}^b + V(\varphi) \\ p &= \frac{1}{2} G_{ab} \dot{\varphi}^a \dot{\varphi}^b - V(\varphi) \\ \frac{D\dot{\varphi}^a}{dt} &= \ddot{\varphi}^a + \Gamma_{bc}^a(\varphi) \dot{\varphi}^b \dot{\varphi}^c. \end{aligned} \quad (4.5)$$

Geometrically, the vanishing of $D\dot{\varphi}^a/dt$ is equivalent to the statement that $\varphi(t)$ is an affinely-parameterized geodesic of the target-space metric, G_{ab} .

For instance, for the $SO(3) \rightarrow SO(2)$ example the $SO(3)$ -invariant metric has the following nonzero Christoffel symbols:

$$\Gamma_{\phi\phi}^\theta = -\sin\theta \cos\theta, \quad \Gamma_{\theta\phi}^\phi = \Gamma_{\phi\theta}^\phi = \cot\theta. \quad (4.6)$$

The geodesics of this metric are the ‘great circles’, corresponding to the intersection of the sphere $S_2 = SO(3)/SO(2)$, with a plane which passes through the circle’s centre.

Once the $SO(3)$ symmetry is explicitly broken (with the $SO(2)$ unbroken), the symmetry-breaking terms of eq. (2.4) imply changes to the target-space connection, leading to the more general expressions

$$\Gamma_{\phi\phi}^\theta = -\frac{G'}{2}, \quad \Gamma_{\theta\phi}^\phi = \frac{G'}{2G}, \quad (4.7)$$

where $G' = dG/d\theta$ and all other components are unchanged. These expressions reduce to eqs. (4.6) given the $SO(3)$ -invariant choice $G = \sin^2\theta$.

The qualitative behaviour of the solutions to these equations is easy to state in the case where the initial scalar kinetic energy, K_i , is large compared with its initial potential energy, V_i . In this case the scalar potential is initially negligible and the scalar moves along the target-space geodesic determined by its initial position and velocity. As it so moves the scalar experiences Hubble friction, which causes it to move more and more slowly along this geodesic with ever-decreasing kinetic energy. Eventually its kinetic energy is similar in size to its potential energy, and so the scalar makes a transition into a potential-dominated regime. At this point the scalar begins to follow the gradients of the scalar potential, until it eventually comes to rest at one of the potential’s local minima. A slow roll occurs if this potential-dominated motion is sufficiently slow. Because for slow scalar motion both the $\ddot{\varphi}^a$ and the $\Gamma_{bc}^a \dot{\varphi}^b \dot{\varphi}^c$ terms in the scalar field equation are small, the entire covariant derivative $D\dot{\varphi}^a/dt$ may be neglected during the slow roll.

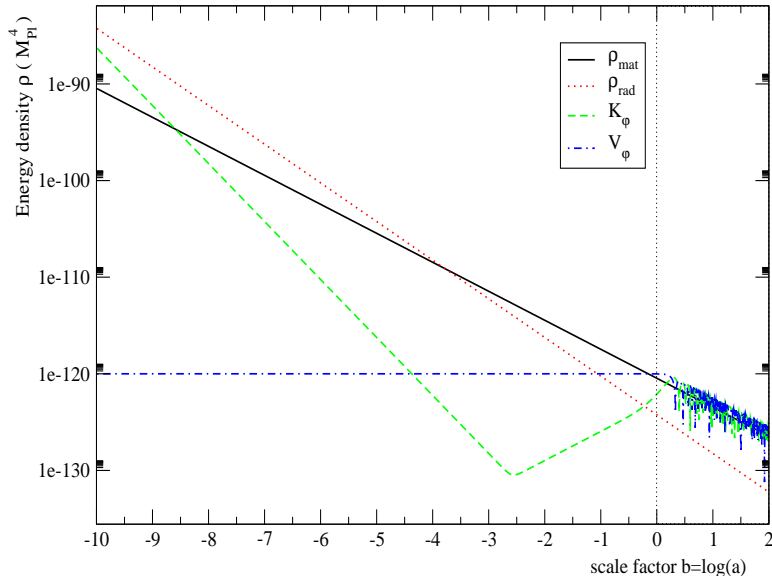


Figure 1: Energy density evolution in the viable quintessence cosmology discussed in the text. Plotted are the energy density in radiation (red dotted), matter (black line), total scalar potential (blue dashed-dotted) and total scalar kinetic (green dashed). Nucleosynthesis occurs at the earliest epoch shown, and the vertical line indicates the present epoch.

4.2 Quintessence Cosmologies

We now examine in more detail the implications of these equations for applications to present-epoch (quintessence) cosmology. We use for these purposes the $SO(3) \rightarrow SO(2)$ pseudo-Goldstone model of the previous section. Besides verifying that such cosmologies can be viable, even after the advent of the WMAP measurements, this exercise is also useful for identifying those features of the resulting cosmologies which might be used to distinguish them observationally from other extant proposals.

For concreteness we have explored the model given by eqs. (2.4), with the choices $f = M_p$, $a = b_4 = \mu^4 = \frac{1}{2}(10^{-30} M_p)^4$, and $b_2 = b_3 = 0$. (These arbitrary choices for b_2 and b_3 are made to arrange minima for the potential at $\theta = \frac{\pi}{4}$ and $\frac{3\pi}{4}$, and maxima of the potential at $\theta = 0, \frac{\pi}{2}$ and π . The main features of the cosmology we present do not depend on these particular details. a is chosen to make $V = 0$ at its minima, and this *is* important for the later cosmology. We have no new insights on the cosmological constant problem in this paper.) Motivated by the simplest power-counting estimates we also choose $c_n = 0$, although we return to this choice at the end of this section, where we also show how our results vary if $c = c_2$ is nonzero.

Fig. (1) shows the results of a numerical evolution of the field equations for this model, giving the evolution of the energy density in radiation and matter, as well as the total kinetic and potential energy density associated with the scalar field motion.

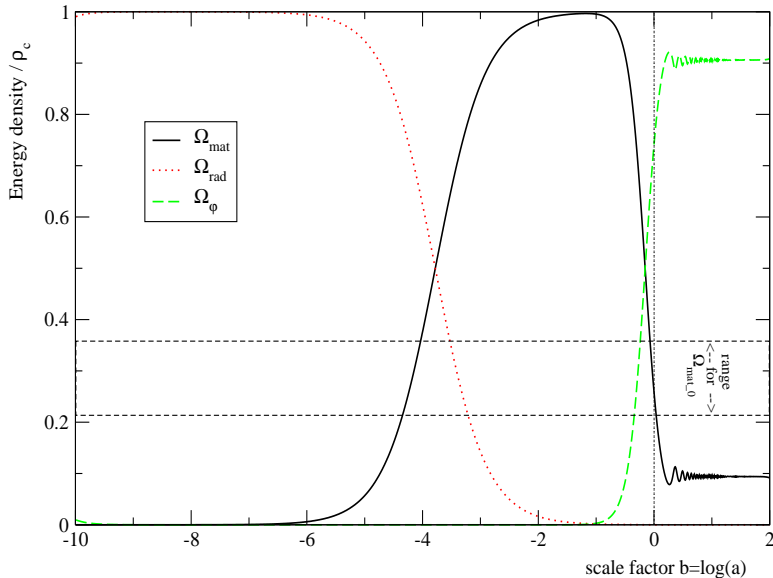


Figure 2: The energy density in radiation (red dotted), matter (black line) and scalars (green dashed) for the same cosmology as the previous figure, given as a fraction of the critical density. The horizontal band indicates the observationally-allowed range for the present-day matter density.

As this figure shows, the scalar fields in this model are just now entering a period of classical oscillation about the bottom of their potential, with the total scalar energy density falling like $1/a^3$ as it is inter-converted back and forth between kinetic and potential energy. Although it is at first sight tempting to place the present epoch during the last period during which Ω does not vary appreciably, this option is disfavoured by its predictions for the equation-of-state parameter $w = p/\rho$, as may be seen from fig. (3).

For the cosmology which these figures illustrate, the initial conditions for the fields θ and ϕ were chosen at the epoch of nucleosynthesis, with θ_0 near $\pi/2$. The initial velocities were chosen so that the initial scalar energy is comparable to the energy in matter and radiation, and since this is much larger than $V(\theta, \phi)$ this means the scalar motion is initially dominated by its kinetic energy. Since the success of standard BBN does not permit the scalar to carry more than 10% of the total energy density, we choose the initial scalar velocities so that $K_\varphi = K_\theta + K_\phi$ saturates this upper bound, with $\dot{\theta}$ initially zero. Here $K_\theta = \frac{1}{2} f^2 \dot{\theta}^2$ and $K_\phi = \frac{1}{2} f^2 G(\theta) \dot{\phi}^2$.

The evolution of the two fields θ and ϕ given these initial assumptions are then shown in fig. (4). This figure shows that the ϕ evolution is quickly damped by Hubble friction. Since the scalar potential has maxima for $\theta = 0$ and $\pi/2$ and minima for $\theta = \pi/4$ and $3\pi/4$, the initial choice $\theta_0 \approx \pi/2$ is close to a maximum. Once Hubble damping reduces the kinetic energy of the scalars to close to their potential energy,

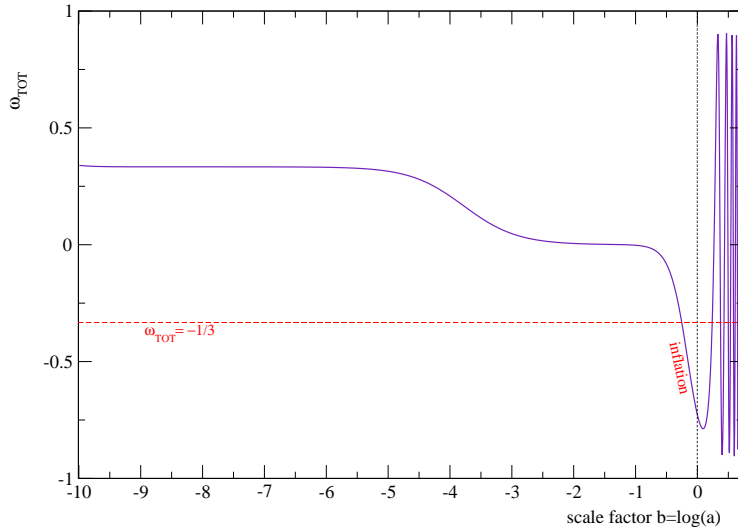


Figure 3: Evolution of the equation of state parameter, $w = p/\rho$, in the cosmology of the previous figures. The horizontal line marks $w = -1/3$, below which the universe accelerates.

θ starts to roll off of its maximum towards the minimum near $\theta = 3\pi/4$. It is striking that neither scalar evolves very far, even though their motion is kinetic-energy dominated for much of the Universe’s history. This feature of the scalar motion may be understood analytically (see appendix), and is a consequence of the extreme over-damping due to Hubble friction.

Characteristic Features

Two features of the scalar motion in this model are generic to quintessence applications of pseudo-Goldstone boson cosmologies.

Late-Time Oscillatory Cosmology: A generic feature of pGB quintessence cosmologies is the late-time oscillations of the scalar fields about the potential’s minimum. (See, however, ref. [12], for a model which differs from most in its late-time consequences.) As is clear from fig. (1), although these oscillations are damped they are not damped faster than the energy density in matter. As a result the Universe settles down into a comparatively steady state, for which the relative proportion of energy tied up in Dark Matter and Dark Energy does not change.

As may be seen from fig (3), these residual scalar oscillations may have observational implications because of the time dependence which they imply for p/ρ , and so also for the acceleration of the Universe. The late-time alternation between acceleration and deceleration is very different from both the eternal or temporary inflation predicted by a cosmological constant or by quintessence based on near-exponential

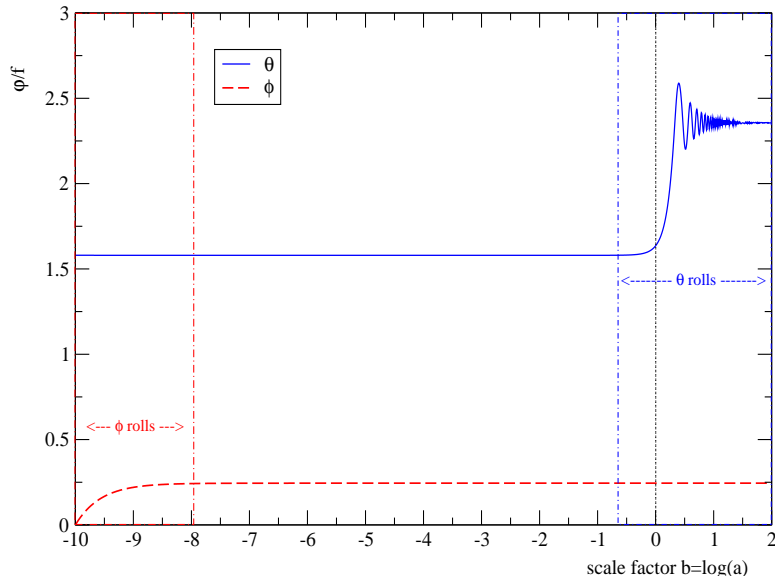


Figure 4: Evolution of the two scalar fields, θ and ϕ , with the initial condition $\dot{\theta}_{BBN} = 0$.

potentials [5, 20, 21, 22], although it is not clear that this would be observable in the foreseeable future.

Special Initial Conditions: A second generic feature of these pGB quintessence models is their sensitivity to initial conditions. Schematically this sensitivity arises because a successful cosmology requires the scalar to be near the maximum of its potential once its kinetic energy becomes comparable with its potential energy. This ensures the Universe experiences a sufficiently long period of potential-dominated slow roll before finally coming to rest at the potential’s minimum.

To quantify how broad a class of initial conditions are acceptable as descriptions of the present-day Universe, we evolved the cosmology described above for a variety of choices for θ_0 and initial scalar velocities and asked which choices satisfied the two WMAP constraints [3]

$$\frac{K_\phi}{\rho_{\text{tot}}} \approx \frac{\rho_\phi}{\rho_{\text{tot}}} = \Omega_\phi = 0.73 \pm 0.09, \quad w = \frac{p_{\text{tot}}}{\rho_{\text{tot}}} < -0.78, \quad (4.8)$$

during the present epoch. Choosing always $K_\phi = K_\theta + K_\phi$ to be fixed at 10% of the total energy density at nucleosynthesis, we varied the distribution of initial energy between the two fields θ and ϕ by varying the parameter $\tan^2 \chi = K_\phi/K_\theta$. Fig. (5) shows the region in the initial $\chi - \theta$ plane which satisfy the two constraints given above. As the figure shows, the allowed region represents a minor fraction of the area of this plane, but is also not infinitesimally small.

The shape of the allowed region is easily understood as follows. It passes through the point $(\chi, \theta) = (\frac{\pi}{2}, \frac{\pi}{2})$ because $\chi = \frac{\pi}{2}$ corresponds to starting with $\dot{\theta} = 0$, and $\theta = \frac{\pi}{2}$

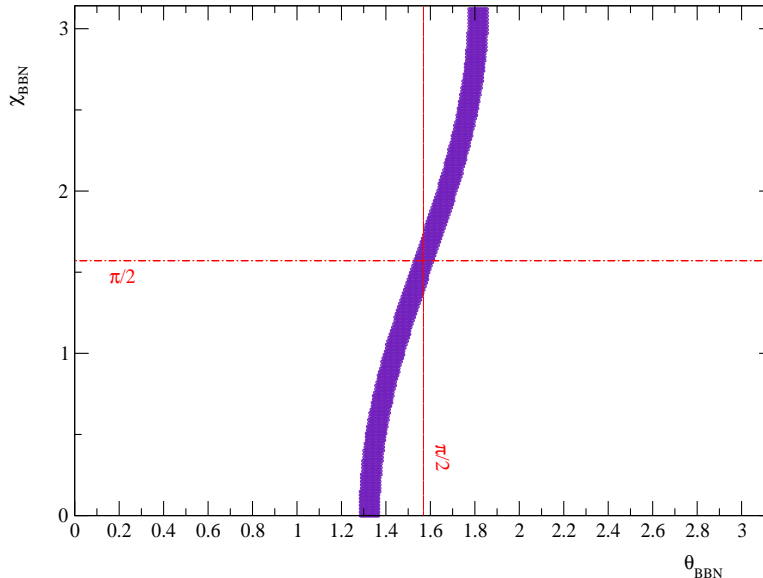


Figure 5: Region of initial conditions in the $(\theta_{BBN} - \chi_{BBN})$ plane which give observationally acceptable values for Ω_φ and w_{tot} .

is the maximum of the scalar potential. The corresponding cosmology simply has θ remain very nearly at rest at the very top of the potential from BBN until now. The curve bends away from $\theta = \frac{\pi}{2}$ as χ varies because if it starts with an initial velocity, θ need not begin at the maximum at BBN in order to end up there during the present epoch.

Sensitivity to G -noninvariant metrics

We close with a discussion of the sensitivity of the above results to the choice of an $SO(3)$ -invariant target-space metric. To test this sensitivity, we repeated the above analyses with the parameter c of eqs. (2.4) nonzero. As expected, we find that c does not change the scalar cosmology unless c is quite large. For instance, fig. 6 shows the range of initial conditions which give acceptable present-day cosmologies if $c = 10$. As is seen from this figure, the allowed region changes perceptibly relative to the $c = 0$ case, but the total acceptable volume does not change (as might be expected from Liouville's theorem).

5. Conclusions

In this paper we re-examine the cosmological applications of pseudo-Goldstone bosons, with the following results.

1. We examine, in §2, the constraints which inflation and cosmology impose on a slowly-rolling scalar field, and reproduce there standard constraints which are

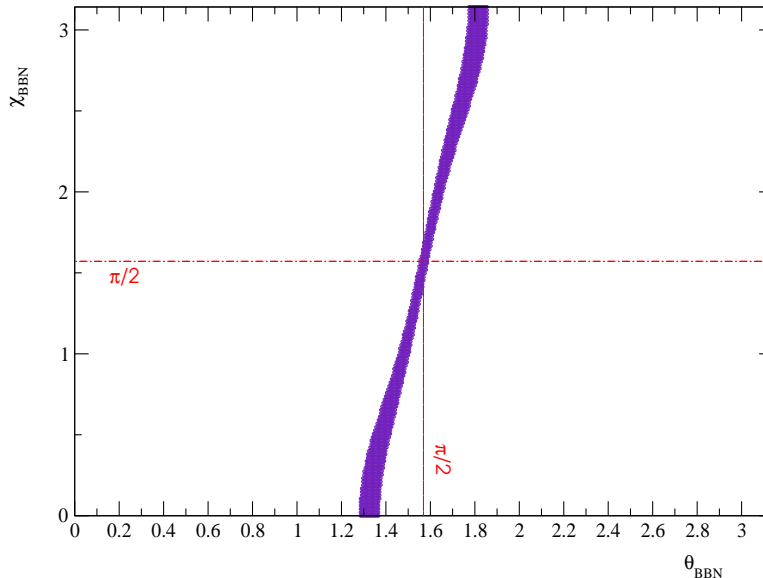


Figure 6: The same analysis as for the previous figure, but with a non-invariant target-space metric (with symmetry-breaking parameter $c = 10$).

implied for the scale f associated with the scalar’s kinetic energy and the scale μ related to its potential energy. These typically require $f \gtrsim M_p$ and $\mu \ll M_p$.

2. In §3 we identify a new brane-world mechanism for ensuring that the scale μ is exponentially small while keeping $f \gtrsim M_p$. It is accomplished by having a theory with an approximate global symmetry which is broken only by the couplings of a massive bulk field to various branes. In this model the scale μ of the low-energy effective theory below the compactification scale is proportional to $\exp(-Ma)$, where M is the bulk scalar mass and a is the inter-brane separation. Once such a small scale is generated in this way in the low-energy theory it is protected against low-energy radiative corrections by the residual approximately-broken symmetry, in the usual manner for a pseudo-Goldstone boson.
3. Motivated by this mechanism for obtaining extremely small scales, in §4 we reconsider the late-time cosmology of such a pseudo-Goldstone boson, by constructing an explicit quintessence cosmology. Successful cosmologies can be made subject to mildly restrictive choices for the initial conditions which are assumed for the scalars at the epoch of Big Bang nucleosynthesis. We argue that pseudo-Goldstone bosons of this type will be observationally distinguishable from other types of quintessence proposals because of the late-time scalar field oscillations which they generically predict.

The great difficulty in obtaining slowly-rolling scalar fields from realistic microscopic theories of physics poses something of an opportunity given the current observational evidence for two epochs during which the Universe underwent accelerated expansion. The challenge is to identify those few kinds of small-distance physics for which cosmologically acceptable scalar fields are possible. In the past, brane-world models have been very successful in circumventing previously-held naturalness obstacles, and the same may be true for the mechanism illustrated by the brane-world toy model which we propose here. We believe this class of models is sufficiently interesting to merit a more detailed exploration of their observational implications for the CMB.

6. Acknowledgments

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7. Appendix A: Supersymmetric Models

In this appendix we briefly summarize how the simple dimensional estimates of the main text can differ for supersymmetric models.

In $N = 1$ supersymmetry in four dimensions scalars arise in complex pairs, as the partners of spin-1/2 fermions in chiral supermultiplets. Furthermore, supersymmetry also requires the quantity G_{ab} can be put into the particular form [24]

$$G_{ab^*} = \frac{\partial^2 K}{\partial \varphi^a \partial \varphi^{b^*}}, \quad (7.1)$$

for a real function, $K(\varphi, \varphi^*)$, known as the Kähler potential. The scalar potential is similarly given in terms of K and the holomorphic superpotential, $W(\varphi)$ by

$$V = e^{K/M_p^2} \left[G^{ab^*} D_a W (D_b W)^* - 3 \frac{|W|^2}{M_p^2} \right], \quad (7.2)$$

where $D_a W = \partial_a W + \partial_a K W/M_p^2$ and G^{ab^*} denotes the matrix inverse of the target-space metric, eq. (7.1).

Additional restrictions arise for K and W if the scalars are also Goldstone bosons for the symmetry-breaking pattern $G \rightarrow H$ [25]. In particular K must be the Kähler function for an appropriate complexification of the manifold G/H , and W must be independent of the Goldstone bosons and their superpartners. For example, for the

two-sphere example, $S_2 = SO(3)/SO(2)$, considered earlier, if the scalars θ and ϕ are related to one another by supersymmetry, then the metric has the form of eq. (7.1) when it is expressed in terms of the stereographic projection to the complex plane,

$$z(\theta, \phi) = \cot\left(\frac{\theta}{2}\right) e^{i\phi}. \quad (7.3)$$

The Kähler potential in this case is

$$K(z, z^*) = 4f^2 \log(1 + z^*z), \quad (7.4)$$

since with this choice

$$\frac{\partial^2 K}{\partial z \partial z^*} dz dz^* = f^2 (d\theta^2 + \sin^2 \theta d\phi^2). \quad (7.5)$$

For the present purposes, the crucial property of supersymmetric theories is that W is protected by a nonrenormalization theorem [26] and so does not receive corrections to any order in perturbation theory, although K does. In supersymmetric models this implies that if a scalar is initially not in the classical superpotential, it cannot enter in perturbation theory as successive scales are integrated out, so long as these integrations remove particles in entire supermultiplets (as is required if the effective theory is to have the supersymmetric form given above). If the vacuum is supersymmetric then the loop-induced dependence of the scalar potential, V , on a pseudo-Goldstone boson must arise through symmetry-breaking contributions to K rather than W .

Once scales of order the supersymmetry-breaking scale, M_s , are integrated out, however, supersymmetry is less restrictive in what it requires. So if the pseudo-Goldstone boson symmetry-breaking scale satisfies $\mu \ll M_s$, none of the above discussion is particularly relevant and the estimates of the main text apply. If $M_s \ll \mu$, on the other hand, it can happen that corrections to the Kähler function, K — and so also for the target-space metric G_{ab} , can be larger than those for V if these are protected by the nonrenormalization theorems. Consequently supersymmetric suppressions are not likely to be relevant for quintessence cosmologies, although they may be relevant for inflationary models.

8. Appendix B: Over-Damped, Kinetic-Dominated Scalar Rolls

In this appendix we identify the a dependence of the scalar field ψ during a period of kinetic-energy-dominated motion. In particular, we establish the result $d\psi/da \propto a^{-p}$, with $p = 3 - m/2$, used in the main text, and derive an upper limit on the total distance ψ can roll during this kind of motion.

We start by changing the independent variable from t to $b = \ln a$, in which case the derivatives of a field ψ become:

$$\dot{\psi} = \frac{d\psi}{db} \frac{db}{dt} = H\psi' \quad (8.1)$$

$$\ddot{\psi} = H^2\psi'' + H'H\psi' \quad (8.2)$$

where over-dots denote d/dt and primes denote d/db . If we also suppose only a single field rolls (so G_{ab} may be set to unity by performing a field redefinition), then for a kinetic-dominated roll the Klein-Gordon field equation becomes

$$\psi'' + [3 + H'/H]\psi' = 0. \quad (8.3)$$

Assuming the dominant energy density satisfies $\rho_m \propto a^{-m}$, with $m = 3$ or 4 for matter- or radiation-domination, we have $3M_p^2 H^2 \approx \rho_m$ and so $H'/H = -m/2$. Consequently $\psi'' + [3 - m/2]\psi' = 0$, with solution $d\psi/db = \kappa \exp[-(3 - m/2)b]$ with κ a constant. Clearly this establishes $\psi \propto \exp[-(3 - m/2)b] \propto a^{-p}$ with $p = 3 - m/2$, as required.

Given this solution we may also compute how far the field rolls, $\Delta\psi$, in a given amount of universal expansion, with the result

$$\begin{aligned} \Delta\psi &\equiv \psi_f - \psi_i = \kappa \int_{b_i}^{b_f} e^{-(3-n/2)b} db \\ &= \frac{\kappa}{(3 - m/2)} \left[e^{-(3-m/2)b_i} - e^{-(3-m/2)b_f} \right] \\ &= \frac{1}{(3 - m/2)} \left[\left(\frac{d\psi}{db} \right)_i - \left(\frac{d\psi}{db} \right)_f \right]. \end{aligned} \quad (8.4)$$

We see that $\Delta\psi$ is directly related to the change in the derivative, $\psi' = (d\psi/db)$, which in turn can be related to the change in scalar kinetic energy, $K_i = \frac{1}{2}\dot{\psi}^2 = \frac{1}{2}H^2\psi'^2$, between the initial and final times.

Denoting the fraction of energy tied up in the scalar field kinetic energy by $\varepsilon = K/\rho_m$, we have

$$\psi'^2 = \frac{2K}{H^2} = \frac{6M_p^2 K}{\rho_m} \approx 6M_p^2 \varepsilon. \quad (8.5)$$

Combining the above results we obtain the final result

$$\frac{\Delta\psi}{M_p} \approx \frac{\sqrt{6}}{3 - m/2} \left(\sqrt{\varepsilon_i} - \sqrt{\varepsilon_f} \right). \quad (8.6)$$

This last expression is useful if the fraction of scalar energy is known or bounded at the initial and/or final times. For instance, since constraints from nucleosynthesis require $\varepsilon_{BBN} \lesssim 0.1$, this is a useful place to choose as the initial or final time. A similar observation has also been made in another context in ref. [23].

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